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WITH THE ASSISTANCE OF

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THE ANNUAL MEETING IN CLEVELAND

The thirty-seventh Annual Meeting of the American Mathematical Society was held at Cleveland, Ohio, from Monday to Wednesday, December 29–31, 1930. The meeting opened on Monday afternoon with a joint session of the Society, the American Statistical Association, and Sections A and K of the American Association for the Advancement of Science. The Council met at the Hotel Statler on December 30 and 31. Since there was no quorum of the Board of Trustees at the meeting called for December 29, there was an adjourned meeting in New York City on January 2. The scientific sessions were held in the building of the School of Education of the Western Reserve University. Tuesday and Wednesday mornings were devoted to sectional sessions of the Society, and on Tuesday afternoon there was a general session to hear an address by Professor G. D. Birkhoff on *Poincaré's Last geometric theorem, its generalizations and dynamical applications.* Later in the afternoon the eighth Josiah Willard Gibbs Lecture under the joint auspices of the Society and the American Association for the Advancement of Science was delivered by Professor Edwin Bidwell Wilson. His address was entitled *Reminiscences of Gibbs by a student and colleague.* On Wednesday afternoon there was a joint meeting of Section A of the American Association for the Advancement of Science, the Mathematical Association of America, and the Society, at which Professor E. T. Bell gave his retiring address as chairman of Section A on *Mathematics and speculation.* Professor Oystein Ore spoke on *Recent developments in abstract algebra* and Professor Karl Menger on *Axiomatic theory of dimension.* At the dinner on Wednesday evening at the Hotel Statler, Professor E. R. Hedrick was toastmaster, and speeches were delivered by Professor J. W. Young, Professor E. T. Bell, and President Eisenhart. A unanimous vote was passed thanking the Presidents and Boards of Trustees of Western Reserve University and the Case School of Applied Science, Dean T. M. Focke, Professor W. G. Simon, and others who had contributed to the success of the local arrangements for this meeting.

The attendance included the following two hundred twenty members:

C. R. Adams, R. P. Agnew, O. P. Akers, J. W. Alexander, E. S. Allen, Mildred Allen, R. B. Allen, N. L. Anderson, R. C. Archibald, C. S. Atchison, W. L. Ayres, R. W. Babcock, C. L. Bacon, G. M. Bareis, I. A. Barnett, Walter Bartky, M. A. Basoco, W. D. Baten, Samuel Beatty, E. T. Bell, H. A. Bender, Theodore Bennett, T. C. Benton, H. R. Beveridge, G. D. Birkhoff, Henry Blumberg, M. G. Boyce, J. B. Brandeberry, R. W. Brink, O. E. Brown, R. S. Burington, W. D. Cairns, B. H. Camp, C. C. Car[illegible]. A. Campbell, R. D. Carmichael, W. M. Carruth, W. B. Carver, Jacques Chapelon, E. W. Chittenden, E. H. Clarke, A. B. Coble, A. Cohen, J. T. Colpitts, A. H. Copeland, E. B. Cowley, W. H. Cramblet, C. M. Cramlet, A. R. Crathorne, C. H. Currier, H. B. Curry, D. R. Curtiss, E. H. Cutler, Wayne Dancer, Tobias Dantzig, L. A. V. DeCleene, E. L. Dodd, H. L. Dorwart, B. F. Dostal, P. S. Dwyer, L. A. Dye, P. D. Edwards, L. P. Eisenhart, J. D. Elder, G. C. Evans, Peter Field, B. F. Finkel, Arne Fisher, W. W. Flexner, T. M. Focke, F. A. Foraker, W. B. Ford, Tomlinson Fort, R. A. K. Frisch, T. C. Fry, W. M. Gafafer, J. J. Gergen, F. J. Gerst, R. E. Gilman, J. W. Glover, L. M. Graves, L. W. Griffiths, V. G. Grove, Harris Hancock, W. L. Hart, C. B. Hennel, E. R. Hedrick, D. M. Hickey, H. C. Hicks, E. H. Hildebrandt, T. H. Hildebrandt, Einar Hille, E. J. Hirschler, B. P. Hoover, H. M. Hosford, Harold Hotelling, J. C. Hughes, W. A. Hurwitz, C. A. Hutchinson, Louis Ingold, M. H. Ingraham, Dunham Jackson, R. P. Johnson, F. E. Johnston, L. S. Johnston, F. C. Jonah, B. W. Jones, A. E. Kennelly, B. F. Kimball, H. W. Kuhn, K. W. Lamson, E. P. Lane, Lincoln LaPaz, G. A. Larew, H. D. Larsen, C. G. Latimer, D. H. Leavens, Solomon Lefschetz, A. D. Lewis, F. P. Lewis, Marie Litzinger, W. R. Longley, Dorothy McCoy, N. H. McCoy, E. J. McShane, W. D. MacMillan, W. O. Menge, Karl Menger, G. M. Merriman, W. M. Miller, E. C. Molina, C. N. Moore, R. L. Moore, T. W. Moore, W. L. Moore, J. C. Morehead, C. C. Morris, Max Morris, Paul Muehlman, F. D. Murnaghan, J. R. Musselman, J. H. Neelley, J. A. Nyswander, E. G. Olds, H. L. Olson, Oystein Ore, F. W. Owens, H. D. Owens, L. J. Paradiso, R. S. Park, B. C. Patterson, H. H. Pixley, L. C. Plant, T. M. Putnam, G. Y. Rainich, Tibor Radó, S. E. Rasor, T. H. Rawles, S. W. Reaves, M. S. Rees, W. T. Reid, C. N. Reynolds, Jr., C. E. Rhodes, R. G. D. Richardson, P. R. Rider, H. L. Rietz, H. P. Robertson, C. F. Roos, S. A. Rowland, L. L. Runge, George Sauté, H. E. Schoonmaker, G. T. Sellew, F. R. Sharpe, H. C. Shaub, W. F. Shenton, L. S. Shively, C. A. Shook, Grace Shover, W. G. Simon, L. G. Simons, H. E. Slaught, L. L. Smail, T. L. Smith, A. E. Staniland, Guy Stevenson, K. D. Swartzel, P. M. Swingle, J. L. Synge, J. D. Tamarkin, J. H. Taylor, J. S. Taylor, C. F. Thomas, J. M. Thomas, M. O. Tripp, B. M. Turner, H. W. Tyler, M. E. G. Waddell, W. J. Wagner, G. W. Walker, J. H. Weaver, F. M. Weida, M. J. Weiss, Willis Whited, J. H. C. Whitehead, E. A. Whitman, G. T. Whyburn, D. V. Widder, Norbert Wiener, R. L. Wilder, K. P. Williams, W. L. G. Williams, C. O. Williamson, E. B. Wilson, F. L. Wren, Kathryn Wyant, B. F. Yanney, C. H. Yeaton, Jacob Yerushalmy, J. W. Young.

The following sixteen persons were elected to membership in the Society:

Professor William Ernest Fred Appuhn, Stevens Institute;
Mr. Kenneth E. Bisshopp, Beloit, Wisconsin;
Mr. Archie Blake, Chicago;
Mr. Hendrik Wade Bode, Bell Telephone Laboratories, New York;
Professor Arley T. Curlee, Salem College;
Mr. Robert L. Dietzold, Bell Telephone Laboratories, New York;
Mr. Bion Hubbard Francis, Bell Telephone Laboratories, New York;
Dr. Jakob Kevitzki, Yale University;
Professor Karl Menger, University of Vienna;
Dr. George Colin Munro, University of Buffalo;
Professor Tibor Radó, Ohio State University;
Mr. William Sell, University of Alabama;
Mr. George Richard Trott, Alabama Polytechnic Institute;
Miss Helen Martha Wright, Columbia University.

As Nominee of Members of the Department of Mathematics of Brown University: Mr. Max Astrachan;

As Nominee of the Missouri State Life Insurance Company: Mr. E. L. Faith;

The Secretary announced that the following members of the London Mathematical Society had entered the Society under the reciprocity agreement.

Dr. Lancelot Stephen Bosanquet, Lecturer in Mathematics, University College, London, England;
Professor Peter Fraser, University of Bristol, Bristol, England.

The ordinary membership in the Society is now 1,926, including 136 nominees of sustaining members and 83 life members. There are also 29 sustaining members. The total attendance of members at all meetings in 1931 was 953; the number of papers read was 386. The number of members attending at least one meeting was 638. The vote for the election of officers closed at the annual business meeting on December 31. There were 284 votes cast and the following officers were elected:

President, Professor L. P. Eisenhart.

Vice-President, Professor W. A. Manning.

Associate Secretary, Professor Tomlinson Fort.

Member of the Editorial Committee of the Bulletin, Professor E. R. Hedrick.

Member of the Editorial Committee of the Transactions, Professor R. D. Carmichael.

Member of the Editorial Committee of the Colloquium Publications, Professor E. T. Bell.

Member of the Editorial Committee of the American Journal of Mathematics, Professor G. C. Evans.

Members of the Council, Professors Marston Morse, James Pierpont, J. D. Tamarkin, K. P. Williams, W. A. Wilson, to serve three years; Professor Warren Weaver, to serve two years.

Members of the Board of Trustees, Professors L. P. Eisenhart, W. B. Fite, Mr. Robert Henderson, Professors G. W. Mullins, and R. G. D. Richardson, to serve two years.

The reports of the Treasurer and of the auditors (Professor H. W. Reddick and Mr. J. J. Tanzola) showed a balance of $4,594.27, exclusive of the balances in the Bulletin, Transactions, Colloquium, Journal, Library, and special funds, and of the life membership reserve. The Society's Endowment Fund, of par value $76,000.00, yielded for 1930 a net income of $3522.71; sustaining memberships for the year amounted to $3,335.00. The amount received from sales of the Society's publications was $7,609.37. During the year the special contributions from members to the amount of $1,110.25 were received, which prevented a deficit on this year's business. The trustees adopted a budget for 1931 showing estimated expenditures and receipts as $38,006.77; and $37,111.77, respectively.

The Librarian reported that the Library of the Society now contains 8,133 volumes.

The following appointments were reported: to represent the Society at the Centennial Celebration of the founding of Randolph-Macon College, October 24, 1930, Professor J. J. Luck; to represent the Society at the Seventy-Fifth Anniversary of the founding of the Pennsylvania State College, October 24, 1930, Dr. Willis Whited; as representative on the National Research Council for the period ending June 30, 1933, and to replace Professor R. G. D. Richardson resigned, Professor D. R. Curtiss; as tellers at the annual election of officers, Professor C. R. Adams, Dr. A. A. Albert, and Professor P. A. Smith.

It was announced that Professor J. F. Ritt had accepted the invitation to give the Los Angeles Colloquium and that his subject is *Systems of algebraic differential equations.*

At the meeting of the Council the following appointments were authorized and announced: as an additional member of the committee on increasing the funds of the Society, Professor W. D. Cairns; as the nominating committee, Professors Virgil Snyder (chairman), L. L. Dines, H. J. Ettlinger, H. H. Mitchell, and H. L. Rietz; as the committee on the Award of the Cole

Prize, Professors L. E. Dickson (chairman), H. F. Blichfeldt, and D. N. Lehmer; as the committee on nomination of the Gibbs Lecturer for 1932, Professors R. E. Langer (chairman), W. R. Longley, and F. D. Murnaghan; as the committee on arrangements for the annual meeting of 1931, Professors H. E. Buchanan (chairman), G. C. Evans, W. L. Miser, R. G. D. Richardson, and W. P. Webber; as representative to the committee on notation of the American Institute of Electrical Engineers, Professor E. V. Huntington; as representative on the National Research Council to succeed Professor Julian Coolidge, Professor E. R. Hedrick; as representatives on the Council of the American Association for the Advancement of Science, Professors Louis Ingold and M. H. Ingraham.

A resolution of appreciation of the work of Professor Dresden, who has served the Society for fifteen years in the secretarial capacity was passed by the Council and also at the annual meeting of the Society. It was announced that Professor R. L. Moore had accepted the visiting lectureship for the academic year 1931–32 and that arrangements would be made for these lectures during March and April of 1932. It was announced that Associate Secretary Mark H. Ingraham will be in charge of the preliminary arrangements for these lectures, and that institutions desiring to have Professor Moore visit them on this lecture tour should notify Professor Ingraham as soon as possible.

Future meetings of the Society were fixed as follows: New York City, February 28, April 3–4, October 31; Minneapolis, September 8–10; New Orleans, December 28–31; Berkeley, April 11. A meeting of the Society has already been arranged for April 3–4 in Chicago.

The Ninth Josiah Willard Gibbs Lecture will be given in New Orleans by Professor P. W. Bridgeman of Harvard University.

The papers whose titles appear below were read in six sections. President Hedrick presided at the section on Analysis on Tuesday morning; Professors E. P. Lane and A. B. Coble at the section on Geometry; Professor E. T. Bell at the section on Algebra. On Wednesday morning President Hedrick and Professor Birkhoff presided at the section on Analysis; Professor Solomon Lefschetz at the section on Point Sets and Dr. T. C. Fry at the section on Probabilities. Mr. Jesse Price was intro-

duced by Professor J. A. Nyswander, Professor Stetson by Professor E. W. Chittenden, Mr. Martin by Professor A. D. Michal, Professor Geronimus by Professor M. H. Ingraham, Dr. Seidel by Professor J. L. Walsh, Dr. Starcher by Professor R. D. Carmichael, Mr. Oldenberger by Professor L. M. Graves, Dr. Schoenberg by Professor G. A. Bliss, Mr. Bohnenblust by Professor Einar Hille, Dr. Cauer by Professor Norbert Wiener, Mr. Craig by Professor H. L. Rietz, and Mr. Evelyn and Dr. Linfoot by Professor Oswald Veblen. The papers whose abstract numbers are followed by the letter *t* were read by title.

Section I, Analysis, Tuesday Morning.

1. *Solutions of differential equations in the neighborhood of singular points*, by Mr. Jesse Pierce. (Abstract No. 37–1–37.)

2. *Note on the Green's function of a star-shaped three dimensional region*, by Dr. J. J. Gergen. (Abstract No. 37–1–26.)

3. *Necessary and sufficient conditions for the representation of a function as a Laplace integral*, by Professor D. V. Widder. (Abstract No. 37–1–34.)

4. *Generalization to three dimensions of the Cauchy integral formula*, by Professor G. Y. Rainich. (Abstract No. 37–1–57.)

5. *A new type of Cauchy integral*, by Professor G. C. Evans. (Abstract No. 37–1–70.)

6. *On double orthogonal series*, by Dr. R. P. Agnew. (Abstract No. 37–1–29.)

7. *On the trigonometric expansion of elliptic functions*, by Professor M. A. Basoco. (Abstract No. 37–1–38.)

8. *Lebesgue derivatives*, by Professor J. S. Taylor. (Abstract No. 37–1–59.)

9. *Closure of the set of translations of a function*, by Professor Norbert Wiener. (Abstract No. 37–1–87.)

10. *On double function space*, by Professor H. E. Stelson. (Abstract No. 37–1–102.)

11. *Some general theorems on composition of singularities*, by Dr. W. J. Trjitzinsky. (Abstract No. 37–1–71–t.)

12. *Small oscillations of the neutral helium atom near the equilateral triangle positions*, by Professor H. E. Buchanan. (Abstract No. 36–11–431.)

13. *On the Fourier series expansions of certain Jacobian elliptic functions*, by Professor M. A. Basoco. (Abstract No. 37–1–40–t.)

14. *On certain theta constants*, by Professor M. A. Basoco. (Abstract No. 37–1–39–t.)

15. *Systems of linear integral equations with applications to an invariantive theory of quadratic functional forms in n functions*, by Dr. T. S. Peterson. (Abstract No. 37–1–55–t.)

16. *One-parameter linear functional groups in several functions of two variables*, by Professor A. D. Michal. (Abstract No. 37–1–62–t.)

17. *The introduction of Stieltjes integrals in the transformation theory of functional forms*, by Professor Michal and Mr. R. S. Martin. (Abstract No. 37–1–63–t.)

18. *Note on functional forms quadratic in a function and its first p derivatives*, by Mr. R. S. Martin. (Abstract No. 37–1–64–t.)

19. *Isogonal trajectories in space*, by Professor Edward Kasner. (Abstract No. 37–1–65–t.)

20. *The analogue of periodicity for general Fourier series*, by Professor R. E. Gilman. (Abstract No. 37–1–68–t.)

21. *On orthogonal polynomials*, by Professor J. Geronimus. (Abstract No. 37–1–72–t.)

22. *The existence of rational functions of best approximation*, by Professor J. L. Walsh. (Abstract No. 37–1–74–t.)

23. *Solution of the problem of Plateau for three contours*, by Professor Jesse Douglas. (Abstract No. 37–1–83–t.)

24. *Proof of a surmisal of Hilbert concerning a role of the Dirichlet principle in analysis*, by Professor Jesse Douglas. (Abstract No. 37–1–84–t.)

25. *Minimax and instability configurations of a soap film bounded by two contours*, by Professor Jesse Douglas. (Abstract No. 37–1–85–t.)

26. *Example of a Jordan curve in space in which no finite area can be spanned*, by Professor Jesse Douglas. (Abstract No. 37–1–86–t.)

27. *On hypergeometric functions of two variables*, by Dr. T. H. Rawles. (Abstract No. 37–1–90–t.)

28. *Linear differential equations whose coefficients are of bounded variation*, by Mr. L. J. Paradiso. (Abstract No. 37–1–89–t.)

29. *On the cluster values of Schlicht and bounded analytic functions*, by Dr. W. Seidel. (Abstract No. 37–1–92–t.)

30. *On the correspondence of boundaries in conformal maps*, by Dr. W. Seidel. (Abstract No. 37–1–93–t.)

31. *Oscillation of sequences and functions*, by Professor W. A. Hurwitz. (Abstract No. 37–1–103–t.)

Section II, Geometry, Tuesday Morning.

32. *Concerning the possibility of certain binary quartics being line sections of the plane quartic curve of genus zero and the complete system of independent forms of two binary quartics*, by Professor J. H. Neelley. (Abstract No. 37–1–42.)

33. *The bitangential curve*, by Professor T. R. Hollcroft. (Abstract No. 36–11–403.)

34. *Complete sets of tensors and invariants of systems of linear homogeneous differential equations of the second order*, by Professor C. M. Cramlet. (Abstract No. 37–1–82.)

35. *A system of circles, associated with a comformal transformation*, by Professor Tobias Dantzig. (Abstract No. 37–1–78.)

36. *The transformation E of nets*, by Professor V. G. Grove. (Abstract No. 36–11–412.)

37. *Plane cubic Cremona transformations with coincident fundamental points*, by Professor H. C. Shaub. (Abstract No. 37–1–50.)

38. *Construction of pencils of equianharmonic cubics*, by Dr. Jacob Yerushalmy. (Abstract No. 37–1–61.)

39. *Groups of motions in spaces with distant parallelism*, by Professor H. P. Robertson. (Abstract No. 37–1–51.)

40. *On linear connections*, by Dr. J. H. C. Whitehead. (Abstract No. 37–1–73–t.)

41. *Multivectorial curvature*, by Professor M. S. Knebelman. (Abstract No. 37–1–31–t.)

42. *Near-collineations and Blaschke's Textilemathematik*, by Professor Edward Kasner. (Abstract No. 37–1–66–t.)

43. *Continuous curves which are homogeneous except for a countable infinity of points*, by Dr. T. C. Benton. (Abstract No. 37–1–66–t.)

Section III, Algebra and Number Theory, Tuesday Morning.

44. *On identities arising from solutions of q-difference equations and some interpretations in number theory*, by Dr. G. W. Starcher. (Abstract No. 37–1–27.)

45. *The regularity of genus of positive ternary quadratic forms*, by Professor B. W. Jones. (Abstract No. 37–1–38.)

46. *The class numbers of real cyclotomic fields*. by Professor C. G. Latimer. (Abstract No. 37–1–45.)

47. *Certain universal functions of generalized polygonal numbers*, by Professor L. W. Griffiths. (Abstract No. 36–11–425.)

48. *Ideal multiplication in a rational linear associative algebra*, by Miss Grace Shover and Professor C. C. MacDuffee. (Abstract No. 36–11–424.)

49. *Tactical configurations of rank two*, by Professor R. D. Carmichael. (Abstract No. 36–9–384–t.)

50. *Algebras of certain doubly transitive groups*, by Professor R. D. Carmichael. (Abstract No. 36–9–383.)

51. *The degree of simply transitive primitive substitution groups of class u*, by Professor Marie J. Weiss. (Abstract No. 37–1–46.)

52. *Reduction of linear transformations to canonical forms*, by Mr. W. O. Menge. (Abstract No. 37–1–52.)

53. *On the Hadamard determinant theorem and orthogonal determinants*, by Professor R. E. Gilman. (Abstract No. 37–1–68.)

54. *Compositions of n-way matrices and multilinear forms*, by Mr. Rufus Oldenburger. (Abstract No. 37–1–44.)

55. *On the Wedderburn norm condition for cyclic algebras*, by Dr. A. A. Albert. (Abstract No. 36–11–417–t.)

56. *Note on Diophantine automorphisms*, by Dr. Leonard Carlitz, National Research Fellow. (Abstract No. 37–1–48–t.)

57. *Numerical functions in the ring of polynomials, mod p.*, by Dr. Leonard Carlitz. (Abstract No. 37–1–49–t.)

58. *Parametric solutions of certain Diophantine equations*, by Professor T. A. Pierce. (Abstract No. 37–1–67–t.)

59. *A composition of determinants*, by Professor Arnold Dresden. (Abstract No. 37–1–79–t.)

Section IV, Analysis, Wednesday Morning.

60. *A new theory of parametric problems in the calculus of variations*, by Professor F. L. Wren. (Abstract No. 36–11–428.)

61. *Note on variation problems with prescribed transversality coefficients and extremals*, by Professor Lincoln LaPaz. (Abstract No. 37–1–76.)

62. *On the problem of Lagrange*, by Professor L. M. Graves. (Abstract No. 36–11–422.)

63. *Discontinuous solutions for the anormal case of the Lagrange problem in the calculus of variations*, by Professor Gillie A. Larew. (Abstract No. 36–11–432.)

64. *On boundary value problems associated with double integrals in the calculus of variations*, by Dr. W. T. Reid. (Abstract No. 37–1–56.)

65. *On the interchange of limit and Lebesgue integral for a se-*

quence of functions, by Professor T. H. Hildebrandt. (Abstract No. 37–1–30.)

66. *Solution of bounded variation of a functional equation*, by Mr. L. J. Paradiso. (Abstract No. 37–1–89.)

67. *The minimizing properties of geodesic arcs with conjugate end points*, by Dr. I. Schoenberg, International Research Fellow. (Abstract No. 37–1–94.)

68. *On the absolute convergence problem for Dirichlet's series*, by Mr. H. F. Bohnenblust and Professor Einar Hille. (Abstract No. 37–1–91–t.)

69. *On the absolute convergence problem for Dirichlet's series*, II, by Mr. H. F. Bohnenblust and Professor Einar Hille. (Abstract No. 37–1–95.)

70. *Two theorems on the partitions of numbers*, by Professor W. B. Ford. (Abstract No. 37–1–104–t.)

71. *Electrical networks and bounded functions*, by Dr. W. Cauer. (Abstract No. 37–3–114–t.)

72. *A transformation of the problems of Lagrange*, by Professor L. M. Graves. (Abstract No. 36–11–423–t.)

73. *A reduction of sufficient conditions in the fixed end point problem of Lagrange*, by Professor Marston Morse. (Abstract No. 37–1–88–t.)

Section V, Theory of Point Sets, Wednesday Morning.

74. *Biconnected and related sets*, by Dr. P. M. Swingle. (Abstract No. 37–1–54.)

75. *On conical accessibility in E_3 (three-dimensional euclidean space)*, by Dr. T. C. Benton. (Abstract No. 37–1–80.)

76. *Spaces in which there exist consecutive points*, by Professor R. L. Moore. (Abstract No. 37–1–32.)

77. *Concerning the sum of a monotonic increasing sequence of continuous curves*, by Professor G. T. Whyburn. (Abstract No. 37–1–77–t.)

78. *Concerning the structure of perfect continuous curves*, by Professor G. T. Whyburn. (Abstract No. 37–1–99.)

79. *A plane, arcwise connected and connected im kleinen set which is not strongly connected im kleinen*, by Professor R. L. Wilder. (Abstract No. 36–11–418.)

80. *The Poincaré duality theorem for topological manifolds*, by Dr. W. W. Flexner. (Abstract No. 37–1–75.)

81. *On a problem of N. Aronszajn and an axiom of R. L. Moore*, by Dr. Leo Zippin. (Abstract No. 37–1–36–t.)

82. *Cyclicly connected continuous curves and correspondences*, by Dr. V. W. Adkisson. (Abstract No. 37–1–41–t.)

83. *Concerning locally connected spaces*, by Professor G. T. Whyburn. (Abstract No. 37–1–58–t.)

84. *On the relative cyclic-element decomposition of subcontinua of a continuous curve*, by Professor G. T. Whyburn. (Abstract No. 37–1–97–t.)

85. *Concerning the "closed components" of the non-regular parts of a continuum*, by Professor G. T. Whyburn. (Abstract No. 37–1–98–t.)

86. *On the cyclic and higher connectivity of locally connected spaces*, by Professor G. T. Whyburn. (Abstract No. 37–1–100–t.)

87. *A c-point connected continuous curve containing no uncountable collection of mutually exclusive continua*, by Professor G. T. Whyburn. (Abstract No. 37–1–101–t.)

Section VI, Theory of Probability and Other Topics, Wednesday Morning.

88. *A method of classification for the distributions obtained in small samples*, by Professor E. G. Olds. (Abstract No. 37–1–33.)

89. *On the distribution of certain statistics obtained from small random samples*, by Mr. A. T. Craig. (Abstract No. 37–1–53.)

90. *Distribution of the correlation coefficient in small samples from non-normal universes*, by Professor P. R. Rider. (Abstract No. 37–1–35.)

91. *A new definition of a Stieltjes integral*, by Professor A. H. Copeland. (Abstract No. 37–1–43.)

92. *Euler-Maclaurin summation formula for two variables by use of Bernoulli polynomials in two variables*, by Dr. W. D. Baten. (Abstract No. 37–1–47.)

93. *Matrices of integers ordering derivatives*, by Professor J. M. Thomas. (Abstract No. 36–11–421.)

94. *Apparent variables from the standpoint of combinatory logic*, by Professor H. B. Curry. (Abstract No. 37–1–60.)

95. *On a problem in the additive theory of numbers*, by Mr. C. J. A. Evelyn and Dr. E. H. Linfoot. (Abstract No. 37–1–96–t.)

M. H. INGRAHAM, *Associate Secretary*

THE SUMMER SESSION FOR TEACHERS OF MATHEMATICS

One of the sessions of the Summer School for Engineering Teachers, held annually by the Society for the Promotion of Engineering Education, will be devoted this year to the study of the teaching of collegiate mathematics.

This Summer School, a unique undertaking in the field of higher education, was established in 1927 as one of the outgrowths of a general investigation of engineering education begun by the S. P. E. E. in 1924. Sessions are held annually in various institutions throughout the country, and have as their purpose the intensive study and discussion by teachers of methods of teaching particular portions of the engineering curriculum. The sessions held thus far have dealt with Engineering Mechanics, Physics, Electrical Engineering, Mechanical Engineering, Engineering Drawing and Descriptive Geometry, and Civil Engineering.

The session this summer, the first to deal with mathematics, will be held at the University of Minnesota immediately preceding the usual summer meetings of the Mathematical Association of America and the American Mathematical Society; it will open on August 24 and will continue for two weeks, closing on September 5. All the usual courses, from trigonometry to differential equations, will be considered in detail.

The teaching staff of the session is headed by Dean O. M. Leland, College of Engineering and Architecture, University of Minnesota, who will serve as Director of the Session, and by Professor C. A. Herrick of the University of Minnesota, who will act as the Secretary. Other members of the staff are: F. T. Spaulding, Graduate School of Education, Harvard University; E. R. Hedrick, University of California at Los Angeles; Louis O'Shaughnessy, Virginia Polytechnic Institute; S. P. Timoshenko, University of Michigan; Dunham Jackson, W. E. Brooke, and M. E. Haggerty, University of Minnesota; H. L. Rietz, University of Iowa; W. J. Berry, Polytechnic Institute of Brooklyn; R. C. Archibald, Brown University; E. V. Huntington, Harvard University; J. W. Young, Dartmouth College; Warren Weaver and C. S. Slichter, University of Wisconsin; T. C. Fry, Bell Telephone Laboratories; C. N. Moore, University of Cincinnati; Leigh Page, Yale University.

Attendance at the session is open to teachers of mathematics in general, and particularly to teachers of mathematics to engineering students. The cost of attendance, aside from travelling expenses, is limited to a registration fee of ten dollars, and a charge for room and meals, which will be approximately $35.00 for the two-week period.

Professor H. P. Hammond, of the Polytechnic Institute of Brooklyn, is the general director of the S. P. E. E. Summer Schools. Further information concerning the session can be secured by addressing him at 99 Livingston St., Brooklyn, N. Y.

H. P. HAMMOND

KELLOGG ON POTENTIAL

Foundations of Potential Theory. By O. D. Kellogg. Berlin, Springer, 1929. ix+384 pp.

The book before us is modern in its field of interest, and is nevertheless based entirely on the kind of mathematics with which the advanced undergraduate, or young graduate, is familiar; hence the American student has now an opportunity, if he will substitute a European for an American habit, and buy a book, to be well grounded in the fundamental subject of potential theory. He will thus possess knowledge which not only has contact with much of the past development of the theory of partial differential equations and its applications to "classical" mathematical physics and is most important in its relations to functions of a complex variable, but also continues to introduce him to the ever new in mathematical problems. Suffice it here to mention in the last direction the problem of Plateau, which lately, by the acquisition of new existence theorems, has opened up large domains for investigation.

The range of subject is suggested by the various chapter headings: The force of gravity; Fields of force; The potential; The divergence theorem; Properties of Newtonian potentials at points of free space; Properties of Newtonian potentials at points occupied by masses; Potentials as solutions of Laplace's equation, electrostatics; Harmonic functions; Electric images, Green's function; Sequences of harmonic functions; Fundamental existence theorems; The logarithmic potential. Velocity fields and the equation of continuity are treated with fields of force, the Heine-Borel theorem occurs in connection with the divergence theorem, developments valid at great distances are made essential for the properties of potentials, ellipsoidal coordinates and the potential of the homogeneous ellipsoid are given in connection with potentials as solutions of Laplace's equation (see Jeans's ingenious generalization of such formulas in the problem of equilibrium of rotating fluids!), conformal mapping is treated under the logarithmic potential, and so on. The final section is devoted to the explicit formulas required for the mapping of polygons.

In a subject of this character it cannot be hoped to make a complete exposition. Beyond the fundamentals, the author will limit himself somewhat narrowly, knowing that although he must omit direct treatment of many subjects he will cross them in the line of march in such a way that the student may return to them later. Thus in the chapter on *Sequences of harmonic functions*, Professor Kellogg is able to devote an incidental four pages to development in spherical harmonics. The main line of march here, indeed, throughout the last third of the book, is towards the exploration of harmonic functions defined in arbitrary regions and the Dirichlet problem. This problem, in which the boundary values are assigned continuously, is a central problem for potential theory.

In order to make sound the treatment of integrals over regions and surfaces it is necessary to discuss the nature of the regions and their bounding surfaces with considerable care and detail. Green's and Stokes' theorems are estab-

lished rigorously in terms of the kind of mathematics possessed by the senior college student, and accordingly this student will have a definite tool for future use. We examine then briefly the set of definitions on which this structure is based (see pp. 112, 113), quoting the essential phrases:

A *regular region of space* is a bounded closed region whose boundary is a closed regular surface. A *regular surface* is a set of points consisting of a finite number of regular surface elements, with the following connections:

(a) Two may have in common either a single point (vertex) or a single regular arc (edge), but no other points;

(b) Three or more may have at most vertices in common;

(c) Any two are the first and last of a chain, such that each has an edge in common with the next;

(d) All those having a vertex in common form a chain such that each has an edge, terminating in that vertex, in common with the next; the last may, or may not, have an edge in common with the first.

Edge, vertex, face and closed surface are then defined. A *regular surface element* (p. 105) is a set of points which, for some orientation of the axes, admits a representation

$$z = f(x,y), \qquad (x,y) \text{ in } R,$$

where R is a regular region of the (x, y) plane, and where $f(x, y)$ is continuously differentiable in R. This is enough of the chain of definitions to indicate the character of the proofs. Obviously the division of the surface into regular elements is not unique, and sometimes regular elements must be subdivided in order to satisfy the requirements of the definition of regular surface.

For the examination of the limiting values of potentials and their derivatives, on approach to surfaces on which distributions lie, there is added the restriction on $f(x, y)$ that it shall have continuous partial derivatives of the second order in R. "Tangential" derivatives of potentials of a single layer and potentials of a double layer are related situations. But whereas a restriction (Hölder condition) on the density of a single layer is necessary for the investigation of the tangential derivatives, the corresponding condition is satisfied automatically in the investigation of the limiting values of the potential of a double layer. Hence the author is enabled to approach the traditional boundary value problems with the mere hypothesis of continuity for the densities of the corresponding distributions.

After an introduction which indicates the ideas underlying several of the older treatments, a systematic analysis of these boundary value problems is carried out in terms of integral equations, which are themselves discussed adequately, for bounded, and for certain classes of infinite, kernels. We have thus solutions of the Dirichlet problem, the Neumann problem, and that where $\partial V/\partial n + h(p)V$, $h(p) \geqq 0$, $\not\equiv 0$, is given on the boundary, for regular regions subject to the mentioned condition on $f(x, y)$. We remark parenthetically that there is a considerable difference between this last problem, solvable in terms of a potential of a single layer, and that where are given boundary values of a quantity $h_1(p)\partial V/\partial n + h_2(p)V$, where $h_1(p)$ may vanish but not identically.

The density distributions on the exteriors of conductors, the exterior Dirichlet problem, and the interior Neumann problem are associated with a characteristic value of the parameter of the integral equation.

The restrictions on the boundary are now dropped, and the next twenty pages (315–338) are devoted to a consideration of the Dirichlet problem for general regions, showing how a solution may be associated with an arbitrary set of values given continuously on the frontier, and establishing criteria for boundary points of the region, dependent on the region merely, at which this harmonic function takes on the assigned boundary value. These criteria are given in terms of barrier functions (Lebesgue) and capacity (Wiener). "Given a domain T, and a boundary point q, the function $V(P, q)$ is said to be a barrier for T at the boundary point q if it is continuous and superharmonic in T, if it approaches 0 at q, and if outside of any sphere about q, it has in T a positive lower bound." "A necessary and sufficient condition that the Dirichlet problem for T, and arbitrarily assigned continuous boundary values, is possible, is that a barrier for T exist at every boundary point of T." Boundary points which admit of barriers are called *regular* boundary points. The solution which has been developed by the method of sequences to correspond to the given set of continuous boundary values takes on these boundary values continuously at q if q is regular; hence the importance of the question as to whether or not a boundary point is regular. From the concept of barrier follow immediately several simple sufficient conditions for this approach, established by Poincaré and Zaremba. Wiener's criterion is fully developed.

The author mentions the limitation that "in all questions of uniqueness the hypothesis on the harmonic function that it be bounded is apt to play an essential part," and gives, for contrast, the function $U=x$ in the three-dimensional domain for which $x>0$, as a harmonic function which is not identically zero but takes on the boundary values zero continuously. However, if, at his suggestion, we make an inversion and Kelvin transformation, in order to obtain an example for a bounded region, the illustration does not remain quite the same. In fact, in the unit sphere, the corresponding function is the familiar fraction in the integrand of the Poisson integral, and fails to approach zero at one point on the surface. A similar feature occurs in the plane.

An investigation of discontinuous boundary value problems would throw a good deal of light on such situations. If a function is harmonic and bounded, say in the unit circle $r<1$, it is determined by boundary values assigned *almost everywhere* on the circumference, even if approach is merely along radii. But if the function is not bounded, and outside a certain class, the situation is different. Thus the function

$$U = \frac{\partial}{\partial\theta}\,\frac{1-r^2}{1+r^2-2r\cos\theta}$$

approaches zero along *every* radius, as r approaches 1. It takes on continuously the value zero at every boundary point but one. Even more startling examples may be given. Moreover, in the opinion of the reviewer, discontinuous boundary value problems which refer to distributions of positive and negative mass, finite in total amount, are as important from the point of view of physics

as the continuous ones. But if the discussion of such problems is omitted from the book before us, except for bibliographical references, the reason is obvious and sufficient. It requires a different preparation in analysis from the one which is assumed.

The same remark applies to development of arbitrary functions in terms of various kinds of sets of orthogonal functions. Many facts are given; but many others, as interesting and as important, depend upon a different sort of analysis. The author must have turned the pages of Fatou's famous memoir with regret at the exclusion of what has been a touchstone for modern mathematics.

Enough has been said to show the reader the interesting and accessible character of the book before us. The author is known among mathematicians for his skillful and clear expositions, as well as for his original contributions and scholarship. The exposition here is reinforced by a number of important exercises, some of them given for the sake of review and drill, but many also in order to present interesting facts and illustrations. The work is unusually rich in scholarly bibliographical data.

G. C. EVANS

VOGEL ON EGYPTIAN ARITHMETIC

Die Grundlagen der Aegyptischen Arithmetik in ihrem Zusammenhang mit der 2:n-Tabelle des Papyrus Rhind. By Dr. Kurt Vogel. Munich, Beckstein, 1929. 211 pp.

This book, an inaugural dissertation of the University of Munich, is an important contribution to the history of mathematics. It is of especial interest to American scholars because its appearance is almost simultaneous with that of Chancellor Chace's monumental edition of the Rhind Papyrus, published by the Mathematical Association of America.*

Dr. Vogel's book is divided into two main divisions, first, a general discussion of ancient Egyptian arithmetic (pp. 5–53) and second, the $2:n$ table of the Rhind Papyrus (pp. 53–181). These parts are preceded by a four-page introduction, discussing the place of the Rhind Papyrus in the history of mathematics; and are followed by a resumé of the chief results obtained, pp. 181–195. There are good indexes and a bibliography of 106 titles.

The first part gives a very clear and satisfactory account of Egyptian arithmetic with particular attention to the use of fractions, including an explanation of the technical expressions used in the Rhind Papyrus and also a brief description of ancient Egyptian weights and measures. The methods of the Egyptian calculator are illustrated by frequent examples from the Rhind Papyrus. It is made clear, among other things, that we are justified in crediting him with considerable skill in mental arithmetic, as well as with a thorough grasp of the fundamental processes, including division by a fraction. Finally, Vogel is able to give a definite answer to the question, Did the ancient Egyp-

* Reviewed by D. E. Smith in this Bulletin, vol. 36 (1930), pp. 166–170.

tians have a concept of the *general* fraction? (It is well known that they had no way of *writing* fractions with numerators greater than unity.) The answer, in the affirmative, is well supported by all the material so well marshalled in this book; and moreover the opinion of certain other scholars* who had previously been skeptical on the point can now be quoted as in agreement with the conclusion, which Vogel states thus (p. 185): "Es wird also jetzt von allen Seiten anerkannt, dass der Aegypter den klaren Begriff des allgemeinen Bruches (in dem nicht-komplexen Sinn) gehabt hat."

The second part is even more important, and contains the author's chief original contributions to the subject. He divides the work into 4 chapters, (1) the theory of partial unit fractions for $2/n$, (2) a study of the actual results stated in the Rhind Papyrus, (3) the two exceptional cases, and (4) the probable development of the table in the Rhind Papyrus. The summary and conclusion occupy the last 15 pages.

In the first chapter (pp. 61–103) the separation of $2/n$ into partial unit fractions is considered from the standpoint of the formula

$$\frac{2}{n} = \frac{1}{x} + \left(\frac{1}{K_1 n} + \frac{1}{K_2 n} + \cdots + \frac{1}{K_\nu n}\right)$$

where $1/x$ is called the "principal fraction" and the parenthesis the "remainder." Since the Rhind Papyrus contains no case where $2/n$ is broken up into more than four unit fractions, we have to consider for ν only the values 1, 2, and 3. Doing this, Vogel is able to derive from the formula all the results of the table in the Papyrus (along with a great many others, naturally) with the exception of 2/35, 2/91, and 2/101, the last of which may be disregarded as trivial, being given as

$$\frac{2}{101} = \frac{1}{101} + \frac{1}{202} + \frac{1}{303} + \frac{1}{606},$$

which simply uses the representation of 2 as

$$1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{6},$$

but which is not in accordance with the Egyptian's otherwise universal rule to take his "principal fraction" greater than $1/n$. As his next step, Vogel modifies his procedure to develop the theoretically possible representations on the basis of the "principal fractions" actually used by the Egyptian in the Rhind Papyrus. For a separation of $2/n$ into only *two* unit fractions, of course the choice of $1/x$ determines uniquely the other, or "remainder" fraction, thus.

$$\frac{2}{n} = \frac{1}{x} + \frac{2x - n}{xn},$$

where $2x - n$ must accordingly equal either 1 or a factor of n. This happens in 28 cases out of the 50 in the table. Of the others, it is sufficient to say here that the separation into three or four unit-fractions leads to divisibility considera-

* Particularly, Peet and Neugebauer.

tions of some complication, so that, as Vogel does not fail to mention, it is quite impossible that we have here the actual method by which the Egyptian computed his table. But the discussion is none the less of interest. Two cases indeed ($n=35$ and $n=91$) do not come under the formula just given at all, and so are made the subject of a separate examination.

In the second chapter (pp. 103–157) the actual table of the Rhind Papyrus is carefully analyzed and the results classified in 5 groups (omitting the two exceptional cases just mentioned). Along with this (pp. 115–128) is a copy of the table itself, with valuable critical notes both on the Papyrus and on Peet's edition.* The main question that now remains to be answered is whether there is any discernible logical principle, any system, by which the Egyptian obtained, first his "principal fraction," and then the "remainder", in cases where this was broken up into two or more parts. Vogel accordingly gives us first, in concise fashion, the conclusions as to this fundamental question that have been reached by all the important previous writers, including Eisenlohr, Cantor, Favaro, Griffith, Hultsch, Vetter, Peet, Gunn, Neugebauer, Bobynin, and Gillain.†

Before giving his own definitive answer to the question, Vogel gives in the third chapter (pp. 157–173) an explanation of the "exceptional cases" $n=35$ and $n=91$, for which the partial fractions given are

$$\frac{2}{35}=\frac{1}{30}+\frac{1}{42}, \qquad \frac{2}{91}=\frac{1}{70}+\frac{1}{130}.$$

It is shown that the method can reasonably be used not only in these two, but in all the other more or less doubtful cases.

In the fourth chapter (pp. 173–181) Vogel sums up his argument with the final conclusion, which seems thoroughly justified by the preceding study, that the choice of the "principal fraction" was made more or less arbitrarily from a number of empirically observed possibilities, but that when this had been done a definite method of procedure was always used, which led to a unique result for the "remainder." He then discusses the possible evolution of the table, its purpose, and its influence upon the arithmetic of the Greeks, Arabs, and medieval Europeans.

Altogether this book is an excellent piece of work, done in a scholarly and interesting fashion.

R. B. McCLENON

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* Reviewed by D. E. Smith in this Bulletin, vol. 30 (1924), pp. 557–559.

† References in detail can be found in the *Bibliography of Egyptian Mathematics* by R. C. Archibald, in Chace's edition of the Rhind Papyrus, vol. I, pp. 127–191.

GRAUSTEIN ON GEOMETRY

Introduction to Higher Geometry. By William C. Graustein. New York, The Macmillan Company, 1930. xv+486 pp., 126 figures.

During the last few years a number of excellent texts have appeared in English on analytic geometry of space, all of which either presupposed or treated in a brief and sketchy manner the foundations on which the theory was developed. And in the same time a number of synthetic projective geometries have been issued which adequately cover the field from that point of view.

It is therefore with particular pleasure that geometers welcome the volume under review. While somewhat belated as compared with the other books mentioned, it can now enjoy the double role of both cause and effect in the present re-awakening of interest in algebraic geometry, a field which requires a wider application of various mathematical disciplines than any other for its proper introduction.

For a book of this nature, three questions at once arise: what is its scope, what are its presupposed premises, and what are its methods? The first and second can be answered in a word, while the third question will be considered more in detail. The scope is to fix the foundations for the study of analytic projective geometry, by establishing the coordinate systems in points, lines, and planes, and applying them to an extensive study of entities defined by linear and by quadratic equations, together with a few equations of higher degree. The field is sometimes the real continuum, sometimes complex, the distinction between the two being sharply drawn as occasion arises.

A knowledge of cartesian analytic geometry of two and three dimensions, of elementary properties of determinants and of the theory of equations, and of the processes of the calculus are presupposed, in so far as these subjects are developed in a first college course in each. In fact a very elementary knowledge of determinants is sufficient.

The book begins with a review of determinants and the solution of a system of simultaneous linear equations, bringing in the ideas of a matrix and of linear dependence from a purely algebraic point of view. Then a synthetic definition of projection, of vanishing points and lines, the theorem of perspective triangles and the principle of duality. Homogeneous coordinates are introduced as ratios, first for the plane, from cartesian coordinates, and the line at infinity is carefully adjoined to the euclidean plane. The geometric meaning of linear dependence is now introduced, and an analytic proof of the plane perspective triangle theorem is given. Harmonic section is approached from the metric side, interpreted in terms of positive and negative line segments on a straight line. Line coordinates are defined as the coefficients in the equation of a straight line, and the earlier definition of duality is justified by the identical algebra in point and line coordinates. Cross ratio is introduced metrically, but is at once shown to be invariant. Linear transformations are approached by rigid motions, which are shown to form a group; then follows a systematic treatment of one-dimensional linear transformations and a less extensive one in two dimensions.

Geometry in the complex plane is prettily introduced by means of a figure, slightly distorted, but with consistent algebraic steps based on contradictory premises, which leads to a paradox, a wise pedagogy. The one-dimensional geometry is now continued to include the fixed points, the characteristic, and the concept of involution, with metric applications.

The next two chapters, the finest in the book, now extend the purely projective treatment of linear transformations to two dimensions, including duality, and a sharp distinction between the two subgroups, that of affine transformations, and that of motion.

Conics are defined as loci of points which satisfy a quadratic equation, mostly with real coefficients, and the form depends on the rank of the relatively invariant discriminant. When the first member of the equation is composite, the conic is called degenerate; in the following chapter, a curve defined by the same property is called reducible, while a little later a conic having a double point is called singular, but the first word is most frequently employed. In view of the many applications of conics having a vanishing discriminant, the word degenerate seems particularly unfortunate. The chapters on tangents employ the method of approach along the curve; continuity is assumed throughout. Now follow the usual properties of conics, including the theorems of Pascal and of Brianchon, largely treated synthetically, then projectivity and involution on a conic. The chapter on pencils of conics does not introduce elementary divisors, but does discuss all the possible forms in which two conics may intersect. Poles and polars are brought in rather late, and applied to centers and diameters and foci.

The general theorem that two superposed quadratic involutions on a one-dimensional rational carrier have one pair of common conjugate elements is not mentioned, but it is used repeatedly and a particular proof given in each case.

The part on plane geometry closes with an extensive geometry of the circle; it includes coaxial systems and inversion, with one paragraph on general inversion as to a conic.

The chapter on geometry of three dimensions can now proceed much more rapidly. It includes the ideas of point and plane coordinates, developed without metric ideas, and an outline of the quadric surfaces, their reguli and tangent planes. Stereographic projection of the sphere is shown to belong to the geometry of (metric) inversion. No mention is made of linear systems of quadrics, nor of curves on a quadric. A short introduction to tetracyclic coordinates and to line geometry show the possibility of other interpretations to systems of coordinates.

The book is provided with a bibliography and with an index. In every chapter are lists of examples, which contribute materially to keeping the book lively and interesting. Solutions of the problems are not provided.

The press work and proofreading have been done so well that the book presents a very attractive appearance, and is strikingly free from typographical errors. The work seems a little long, but it would be an embarrassing task to show just what parts should have been omitted.

VIRGIL SNYDER

SHORTER NOTICES

Identität und Wirklichkeit. By Émile Meyerson. Translated from the third French edition by Kurt Grelling; with Introduction and Notes by Leon Lichtenstein. Leipzig, Akademische Verlagsgesellschaft, 1930. 40+534 pp.

The philosophy of Émile Meyerson has been expounded during the last twenty years, in four large volumes, the first of which has now appeared in its third edition; it is this edition which has formed the basis for the German translation that is before us for review. It contains besides the author's preface and three appendices (on *Leibniz, Newton, and action at a distance*; on *The followers of Copernicus and the principle of inertia*; and on *Herakleitos* πάντα ρει), an introduction to the philosophy of Meyerson by Lichtenstein. This introduction makes clear that the work of Meyerson is of especial interest and significance for mathematicians as well as for those concerned with the sciences. A detailed examination of the bearing of Meyerson's work upon the philosophical foundations of mathematics would doubtless be of great value. It must suffice here to say that his fundamental doctrine refers to the conflict between the nature of the human mind and that of nature outside man, and that his method consists in an historical survey of the manner in which scientific conclusions have been arrived at.

ARNOLD DRESDEN

Das Unendliche in der Mathematik und seine Ausschaltung. By Felix Kaufmann. Leipzig and Vienna, Franz Deuticke, 1930. x+203 pp.

This book advocates nothing less than the elimination of the infinite from mathematics. At first this seems a bit startling (though the idea itself is hardly original), but it soon turns out that the author's viewpoint is not very far from that of Brouwer and the intuitionists, on whom he leans quite heavily.

The author's major interest seems to be in philosophy, but one learns from the preface that he has been interested in the foundations of mathematics for thirteen years, and for six years has taken part in the discussions of the foundations undertaken by a circle of philosophers and mathematicians of Vienna under the leadership of M. Schick and H. Hahn. He acknowledges the cooperation of A. Becker, A. Fraenkel, and K. Menger.

The book itself, apart from its thesis, can be recommended as a pleasant introduction to the foundations of mathematics in the present controversial state of that subject. It contains a good account of the positions of Brouwer, Hilbert, Russell, and Fraenkel. Especially welcome is the clear presentation of the intuitionist case. The chapter headings are: Einleitung; I. Grundtatsachen der Erkenntnis; II. Symbolik and Axiomatik; III. Natürliche Zahl und Menge; IV. Negative Zahlen, Brüche, und irrationale Zahlen; V. Die Mengenlehre; VI. Das Problem der durchgängigen Entscheidbarkeit arithmetischen Fragen; and VII. Die Antinomien.

If we accept Brouwer's view, the only sets which exist are those which are countable and have been effectively well-ordered. The author remarks,

and this is his principal point, that we can as well go the whole way and admit the existence only of finite sets. Any statement about a countable, effectively well-ordered set can by a circumlocution be translated into a statement about the rule by which the elements follow each other in the well-ordering, which rule is something finite and definite. For example, we are shown how it is possible in his scheme to prove that every bounded monotonic sequence of irrational numbers has a limit. Of course both the sequence and the irrational numbers composing it must be given effectively, since for the intuitionists only those irrational numbers exist which have been effectively defined.

The author begins the introduction by remarking that just as the nineteenth century witnessed the banishing of the infinitely small from analysis through the work of Weierstrass and others, so he will show that we can (and must) dispense with the actual infinite of Cantor. This analogy is not perfect, however, as there exist quite respectable number systems and geometries with absolute infinitesimals, even if they have no place in the ordinary function theory.

The first two chapters contain the philosophical ideas of Husserl and Brouwer which have led the author to his conclusions. These may be of less interest to the mathematician than the succeeding chapters where these ideas are applied to the outstanding problems of the foundations of mathematics. The third and fourth chapters deal with the foundations of the real number system. The fifth chapter discusses general set theory, ordinal numbers, and the various attempts to put the theory of sets on an axiomatic basis. The sixth chapter raises the question of the "categoricalness" of the system of positive integers. Can every statement about the positive integers be either proved or disproved? The conclusion is reached that at least it can never be proved that the answer to this question itself is "No." The last chapter concerns the paradoxes of the infinite. It is not suprising that these should cease from troubling in a system from which infinite sets are excluded.

It seems to me that the author, besides producing an interesting book, has made a good case for the contention that if we accept Brouwerism, we can get along theoretically without the notion of an infinite set, whether or not that notion is meaningless, as the author maintains. However, I do not believe that the views of Brouwer will ever find general acceptance among mathematicians. As this is not the place for an elaborate discussion of the questions raised by the intuitionists, I should merely like to add the following minor point to the prevailing confusion. If the continuum hypothesis is true, it is conceivable that someone may someday discover an effective way of well-ordering the real number continuum so that every number has only a countable number of predecessors. If this were done it would be practically a refutation of Brouwerism. It might seem, then, that either the intuitionists must prove that it cannot be done, or must proceed with a sort of sword of Damocles hanging over their heads. But it would be a violation of intuitionist principles even to attempt such a proof, to so much as mention the real number continuum. The intuitionist contentions are not themselves based on proof. On the other hand they require that mathematicians be very fussy about their mathematical proofs and avoid the use of reductio ad absurdum arguments and the axiom of choice.

ORRIN FRINK, JR.

Esercizi di Analisi Matematica, by G. Fubini and G. Vivanti. 2d edition. Turin, Società Tipografico, 1930. x+366 pp.

This excellent collection of problems is intended to be used in conjunction with texts on the calculus written for students interested primarily in the engineering applications. It is not a mere list of problems with answers. Each of the ten chapters begins with a concise statement of those parts of the theory on which the problems are based. Of a total of nearly seven hundred problems, about two hundred and fifty are fully discussed in the text; for the others there is occasionally a hint, more frequently a mere statement of the result. Such problem collections are indispensible and very useful because the texts which they are intended to supplement but rarely contain worked-out examples, and much more rarely exercises for the reader. The ground covered by the book corresponds roughly to that treated in the introductory course in calculus and in the courses in advanced calculus and differential equations of our college curricula. It also includes an appendix which contains a short discussion of the problems of the elastic beam and a dozen problems discussed in detail. A student who has worked through this collection of problems should gain therefrom thorough mastery of the field of elementary calculus and differential equations. It is to be recommended strongly to the attention of teachers.

ARNOLD DRESDEN

Lezioni di Analisi Matematica, by Giulio Vivanti. Volume I. Torino, S. Lattes, 1930. viii+479 pp.

The present book was written to supply the need brought about by a reorganization of mathematical instruction in the Italian universities. In the words of the preface "in nearly all Italian Universities, in some by edict and in others by custom, the courses in higher algebra and in infinitesimal analysis have been fused into a single two-year course." Consequently in place of issuing a third edition of his well known *Lezioni di Analisi infinitesimale* (reviewed in this Bulletin, vol. 28 (1922), pp. 315–16), the author has written a new book, presumably in two volumes, the first of which is before us for review.

Of the fifteen chapters which make up this volume, those numbered I, VII, XIV, and XV, and part of chapter VI, altogether about half of the book, are essentially identical with corresponding parts of the second edition of the earlier book. It should be observed that of the changes which have been made, some have been dictated by the insertion of the chapters on algebra, while a few others represent modifications in the method of presentation.

The new chapters are devoted to algebraic topics. The material contained in them corresponds in the main to the content of the American college course in the theory of equations.

This book shows, in many places, the hand of a skilled expositor. It has a remarkably clear style; its proofs are models of presentation. I believe that many teachers of calculus courses could make good use of this book for supplementary reading.

ARNOLD DRESDEN

Foundations of Geometry and Induction. By Jean Nicod, with prefaces by Bertrand Russell and André Lalande. New York, Harcourt, Brace, and Co., 1930. 284 pp.

This book is a posthumous publication of the two theses which its author had completed before his death in 1924. The first part, consisting of about 190 pages, is entitled Geometry in the Sensible World and has a preface by Bertrand Russell; the second, of about 80 pages, which deals with the Logical Problem of Induction, is introduced by André Lalande. After having perused the book the reader will doubtless agree with these two teachers of the author, that he was a man of great promise and that his early death was a distinct loss for the future development of the philosophy of the sciences. It would be unfortunate, however, to consider this work as a mature attainment. It seems to me valuable rather for what it attempts than for what it actually accomplishes.

The second part, which deals with a problem of real concern to the natural sciences rather than to mathematics, is linked very closely to the Theory of Probability of Keynes, with whose position the author agrees in the main. This part of the book will also be of interest to those acquainted with Nicod's earlier paper on *A reduction in the number of primitive propositions of logic* (Proceedings of the Cambridge Philosophical Society, vol. 19), in which he showed how the formal logic of Whitehead and Russell's *Principia* could be based on the single undefined notion of "negation."

The first part represents an elaboration and partially a critique of those sections in Poincaré's *Science et Hypothèse* in which this great scientist sought to establish for geometry a basis in the domain of the senses. We find here constructions in terms of sense data, isomorphic with certain simple geometries; these constructions are more or less elaborate, but in every case they are robbed of their immediate reality by idealizations without which the problem would probably be entirely unmanageable. A clear account of these constructions can not be made without mention of many details; and hence a critical examination of the work must be omitted. The principal purpose of this work becomes clear from the following passage on page 13: "The discernment of the sensory order around us, which forms the qualitative background of our life and of our science, and which is ever present, however indistinctly, should certainly be a source of curiosity to any philosopher, even if his metaphysics should not obtain any aid from it. Such is the end at which we aim. We hope to approach it by the study of the objective aspect of geometry. It is impossible, in fact, to possess a proper idea of the order of our sensations if we are hampered by a false or confused idea of space."

There are unfortunately some things in this book which will cause mathematicians to squirm; such are the discussion of "infinitely small" on page 217, the talk about infinite probability, and a passage like the following on page 21: "The discovery of one system of meanings satisfying a group of axioms is always logically very important; it constitutes the proof that these axioms do not contradict one another."

Finally I must make a remark which applies not only to this book but equally to most other philosophical books which deal with the foundations of mathematics. There are definitions and reasoned discussions; but one is never informed as to the fundamental, undefined concepts nor about the basal as-

sumptions. Consequently one has the sensation of the oft quoted traveler who is aware that he is going, but who does not know his destination. Possibly and probably such discussions are necessary as a preliminary clearing of a field. They give one a feeling for the significance of a question and may help to formulate the essential problems. But they can never be conclusive in character. To obtain real insight in the foundations of mathematics, mathematical methods have to be used.

ARNOLD DRESDEN

Tafeln der Besselschen, Theta-, Kugel-, und anderer Funktionen. By Keiichi Hayashi. Berlin, Julius Springer, 1930. 125 pp.

Fünfstellige Funktionentafeln. By Keiichi Hayashi. Berlin, Julius Springer, 1930. 176 pp.

The title of the first of these volumes indicates its principal contents, the other tables referred to being for the most part subsidiary to those named. An idea of the scale on which this useful compilation has been made can be had from the table for $J_0(x)$ and $J_1(x)$. From 0.000 to 0.110 the entries are carried to sixteen places; from 0.12 to 0.50, to fourteen places; and from 0.50 to 25.10, to twelve places. Among the less usual functions included in this collection are $1/(n!)^2$, $n=1, 2, \cdots, 70$, and the arithmetic-geometric mean between 1 and k', $(k')^2=0.00000$ to 0.00300. The presence of these two tables illustrates the intention of the author to provide the computer with means of amplifying, if necessary, the primary tables of this volume. A convenient appendix of fourteen pages gives the analytic expressions for the functions tabulated.

In *Fünfstellige Funktionentafeln*, we find an even greater variety of tables. Besides abbreviated forms of the tables found in the volume which we have just mentioned, there are included in the forty-nine tables listed such functions as the gamma function, tables of powers and factorials, values of the probability integral as a function of its upper limit, the solutions of eight transcental equations such as $\tan x=x$, and many other items for which the computer will be grateful.

As for the physical make-up of these books it would be sufficient to point out that they are from the press of Julius Springer, were it not for the fact that to users of tables this is matter of special importance. The reviewer wishes to record that he has never seen tables in which the general appearance of the printed page is more pleasing to the eye.

T. H. RAWLES

Statistical Résumé of the Spearman Two-Factor Theory. By Karl J. Holzinger. Chicago, The University of Chicago Press, 1930. 43 pp.

An interesting and clearly expressed review of Spearman's theory that every mental ability may be resolved into two linear components, of which one contains a factor common to all abilities, and the other a factor specific to the given ability. Some original mathematical discussion has been added, and the theory is amply illustrated. Much of the data for the illustrative material was gathered by the author.

B. H. CAMP

Das Prinzip der Kleinsten Wirkung von Leibniz bis zur Gegenwart. By Adolf Kneser. (Vol. IX of the Wissenschaftliche Grundfragen, edited by R. Hönigswald.) Leipzig, Teubner, 1928. 70 pp.

The principle of least action is treated in this little book from an historic and philosophic point of view. While the formulation of the concept "action" varies as it is applied in one field or another, the fact that it is possible to envisage a great diversity of natural phenomena as solutions of problems which ask for the minimum or maximum of the "action integral" presents the philosopher with a remarkable opportunity for speculations concerning unity and harmony in nature. And this opportunity has been eagerly seized. Beginning with an account of the way in which the idea of the least-action principle took root in the work of Leibniz, the author traces its development by Leibniz, Euler, Helmholtz, and Planck. The contributions made by Maupertuis, after whom the principle is frequently named, are put in relation to those of Leibniz and Euler; and the work of the erstwhile president of the Prussian Academy of Sciences, as far as it is concerned with the least-action principle, is not deemed of great value. Besides, it seems that priority for a formulation of the principle belongs to Leibniz rather than to Maupertuis "who indeed never succeeded in reaching an exact formulation of his principle nor in making a single correct application of it."

Of great interest are the connections which the author establishes between this principle and the studies on the bases of science made by various philosophers. There are numerous quotations from Kant's *Critique of Judgment*, to whose ideas on the nature of the cosmos the principle of least action is shown to bear important relations.

If this principle is to be regarded as of general significance in the physical universe, there is opened up an alluring field for speculation. For in its character as a principle of natural economy it is, at least superficially, in striking contrast with the prodigality which characterizes the organic world.

The collection of monographs of which the book under review is one, is a valuable aid in the study of fundamental questions in a wide variety of fields of knowledge.

ARNOLD DRESDEN

Integralgleichungen, unter besonderer Berücksichtigung der Anwendungen. By G. Wiarda. Leipzig and Berlin, B. G. Teubner, 1930. ii+183 pp.

This small book of five chapters is intended as a simple introduction to the theory of integral equations. Its announced purpose is to inspire the interest of the student of pure mathematics and to be of use to the student of applied mathematics. The book treats exclusively the Fredholm equation of the second kind and centers upon the Schmidt theory as most directly adapted to the applications. The exposition is elementary and should be readable to the student versed in the calculus and the fundamentals of the theory of infinite series. The author does not disdain at opportune points to illustrate his deductions by calculations in specific examples.

The compass of the book is small and the treatment accordingly affords few digressions from the central theme into related fields. Even the relation be-

tween integral and differential equations is but briefly treated. This feature may perhaps recommend the book to students who wish merely to dip into the subject. Others will find little here that is new, and yet the reviewer found the book distinctly pleasant to read. The exposition is carefully and skillfully done. Definitions and theorems are conspicuously set forth, and the author has taken pains to keep the reader informed at all times as to his purpose and the proposed means for its accomplishment.

The contents of the book may be briefly summarized. Chapter 1, designed to motivate the study of the integral equation, is given mainly to a rather detailed consideration of the physical problem presented by the stretched string under both free and forced vibration. It includes also the formulation of a problem in optics. Chapter 2 is purely mathematical, and contains a systematic exposition of the fundamental theorems of the Schmidt-Hilbert theory of the equation with a symmetric kernel. Chapter 3, entitled "Applications," considers the problems of the flow of heat in a rod, and of the deflection of a loaded beam. The practical computation of a solution by the method of successive approximations is briefly developed. Chapter 4 is devoted to the equation with an unsymmetric kernel; and finally in Chapter 5 a short outline of the Fredholm theory is given.

To those who would form an acquaintance with this branch of mathematical theory and its applications to physical problems, the book might well be recommended.

R. E. LANGER

Comptes Rendus du Septième Congrès des Mathématiciens Scandinaves tenu à Oslo 19–22 août, 1929. Oslo, A. W. Brögger, 1930. 1919+7 pp.

Comptes Rendus du Premier Congrès des Mathématiciens des Pays Slaves. Warsaw, Ksiaznica Atlas, 1930. 239+4 pp.

The first Scandinavian mathematical congress was held in Stockholm during 1909 on the initiative of Mittag-Leffler. The meetings have since been continued at irregular intervals alternating between the capitals of the four Scandinavian countries. The present congress was held in Norway in connection with the commemoration of the centennial of N. H. Abel's death. The proceedings contain 22 papers, almost all published in international languages. Some interesting expository lectures are also included in the proceedings. The list of authors i cludes: Alander, Ahlfors, Arwin, Brun, Danielson, Engstrom, Fjeldstad, Heegaard, Jessen, Juel, Nagell, F. Nevanlinna, R. Nevanlinna, Nystrom, Ore, Rasch, Rasmussen, Rode, Skolem, Selberg, Tambs-Lyche.

The first congress of mathematicians from the Slavic countries was held in Warsaw in September, 1929. The proceedings contain 54 papers, about half of them in the Polish language. The authors are: Bydzovsky, Chwistek, Dickstein, Dusl, Fréchet, Golab, Hadamard, Härlen, Hoborski, Hosiasson, Hostinsky, Jarnik, Kaczmarz, Karamata, Kawaguchi, Kempisty, Knaster, Kolodziejczyk, Koutsky, Kuratowski, Leja, Lubelski, Mazurkiewicz, Menger, Milicer-Gruzewska, Neyman, Nikodym, Obreschkoff, Petr, Petrovitch, Peyovitch, Popoff, Popova, Presburger, Rychlik, Saltykow, Sergescu, Sierpinski, Sieczka, Slebodzinski, Steinhaus, Szpilrajn, Tschakaloff, Vitali, Wazewski.

OYSTEIN ORE

Théorie des Fonctions Algébriques et de leurs Intégrales. Par P. Appell et E. Goursat. Tome I, *Étude des Fonctions Analytiques sur une Surface de Riemann.* Deuxieme édition, revue et augmentée par P. Fatou. Paris, Gauthier-Villars, 1929. xxxv+526 pp.

As the title indicates, this Tome I is a second edition of Appell and Goursat's *Fonctions Algébriques*, with revisions by the late Fatou. Tome II, which will be reviewed in a later number of this Bulletin, is a new treatise on automorphic functions, composed entirely by Fatou.

The first edition of Appell and Goursat was published in 1895. Everybody is familiar with its beautiful chapters, in which the theory of algebraic functions and abelian integrals is developed with rare skill and consummate style. The second edition closely follows the first. Since there is no preface to explain the changes made, such changes can be determined only by a page to page comparison of the two editions. The most conspicuous modification is an improved treatment, in Chapter VI, of the theorem that every algebraic curve is birationally equivalent to a curve whose only singularities are double points with distinct tangents. There are other differences, but they are not fundamental. Through a slip, the page numbering in the table of contents is that of the first edition.

The republishing of this classic work will assure, for our younger students, the possibility of reading the theory of abelian integrals with the same enjoyment that their teachers derived from the original edition.

J. F. Ritt

NOTES

A new journal, the Rendiconti del Seminario Matematico della Reale Università di Padova, has been founded, under the editorship of A. Comessatti, E. Laura, A. Tonolo, and G. Vitali.

The next International Congress of Applied Mathematics will be held at Cambridge, England, in the Summer of 1934.

The Centenary Meeting of the British Association for the Advancement of Science will be held at London, September 23–30, 1931, under the presidency of General Smuts. Sir J. J. Thomson has been elected president of Section A (mathematical and physical sciences).

The Mexican scientific society "Antonio Alzate" (founded in 1884) has, by virtue of a resolution passed by the President of the Republic and the Department of Public Instruction, been constituted as the National Academy of Sciences, under the title Academia Naçional de Ciencias Antonio Alzate, and was inaugurated December 9, 1930.

At the Cleveland meeting of the American Association for the Advancement of Science, Assistant Professor C. F. Roos, of the department of mathematics of Cornell University, was elected permanent secretary, as successor to Professor B. E. Livingston, who becomes general secretary. Professor E. R. Hedrick was elected vice-president and chairman of Section A (mathematics), and Professor G. C. Evans vice-president and chairman of Section K (social and economic sciences).

At the annual meeting of the Mathematical Association of America, Professor E. T. Bell was elected president, and Professors Arnold Dresden and C. N. Moore vice-presidents.

The Paris Academy of Sciences announces the award of the following prizes for 1930: the Poncelet prize to Arnaud Denjoy, for his mathematical work; the Francœur prize to Eugene Fabry, for his study of the singularities of analytic functions; the Henri de Parville prize to Emile Duchêne, for his work in aerodynamics and ballistics; the Lalande prize to Nicolas Stoyko, for his theoretical and practical work in the calculation of planetary orbits; the Pierre Guzman prize to Alexandre Véronnet, for his work in cosmogony; the Delalande-Guérineau prize to Félix Ollivier, for his work on photogrammetry entitled *La Topographie dans Topographes*; the LaCaze prize to Henri Abraham, for his scientific work; the Hébert prize to Richard Langlois, for his work entitled *Les Machines Asynchrones à Champs Tournants, à Bagues et à Collecteur, Théorie Générale et Applications*; The Montyon prize in statistics to René Roy, for his work in the application of mathematics to statistics and economics; the Binoux prize for history and philosophy of the sciences jointly to Pierre Brunet, for his work entitled *Les Physiciens Hollandais et la Méthode Expérimentale en France au XVIIIe Siècle* and his biography of Maupertuis, and to Niels Nielsen, for his work entitled *Géomètres Français sous la Révolution*; the Jeanbernat-

Doria prize to Henri Volkringer, for his work entitled *Les Étapes de la Physique*; the Grand Prix for mathematical sciences to Georges Valiron, for his work in analytic functions; the LeConte prize to Elie Cartan, for his mathematical work, especially in infinite groups, differential geometry, and integral invariants; the Houllevigue prize to Georges Giraud, for his work in partial differential equations; the Caméré prize to David Wolkowitsch, for his work entitled *Applications de la Géométrie à la Stabilité des Constructions*; the Hirn Foundation prize to Maurice Kraïtchik, for his research and treatises on the theory of numbers.

The scientific prize of the Lasserre Foundation has been awarded to Professor Georges Giraud, of the University of Clermont.

The Ernst Abbe prize of the Carl Zeiss foundation has been awarded to Dr. Philip Furtwängler, professor of mathematics at the University of Vienna.

The Alfred Ackermann-Teubner memorial prize of 1930 has been awarded to Professor Johannes Tropfke, for his *Geschichte der Elementar-Mathematik* and his work on the share of Archimedes in the development of trigonometry.

The Council of the Royal Astronomical Society has awarded its gold medal to Professor W. de Sitter, for his theoretical investigations on the orbits of the satellites of Jupiter and his contributions to the theory of relativity.

The New York Academy of Sciences has awarded one of its A. Cressy Morrison prizes to Professor H. von Zeipel, of the Astronomical Observatory of Upsala, for his paper entitled *The evolution and constitution of stars.*

Emile Jouguet has been elected a member of the Paris Academy of Sciences in the section of mechanics, as successor to the late Hippolyte Sebert.

Professor Emile Picard has received an honorary doctorate from the University of Liége.

Professor Max Planck has been made chancellor of the Prussian Order of Merit for Science and Art.

On the occasion of its seventy-fifth anniversary, the Zurich Technical School has conferred an honorary doctorate on Professor Ludwig Prandtl, of Göttingen.

Sir Ernest Rutherford, professor of experimental physics and director of the Cavendish Laboratory of the University of Cambridge, has been made a baron.

In the Faculty of Sciences of the University of Paris, the following maîtres de conférences have been promoted to professorships: Pauthenier (physics), Beghin and Foch (mechanics of fluids).

Dr. E. Wieselberger, of the Kaiser-Wilhelm Institut für Strömungsforschung at Göttingen, has been appointed professor of applied mathematics at the Aachen Technical School.

Professor Wilhelm Blaschke, of the University of Hamburg, has recently delivered lectures, as visiting lecturer of the American Mathematical Society,

at the following Universities and Colleges: Princeton, January 6, 7; Swarthmore, January 8; Columbia, January 14, 15; Cornell, January 22, 23; Chicago, January 27, 28; Iowa, February 4; Wisconsin, February 11; Ohio State, February 13, 14; Dartmouth, February 16, 17; Massachusetts Institute of Technology, February 19, 20, 24; Harvard, February 25; Brown, February 26. The subjects of these lectures were "Topological Questions of Differential Geomtry," "Selected Topics from Differential Geometry," "Mathematical Problems of Nomography," and "The Geometrical Foundations of the Theory of Relativity." Professor Blaschke will lecture at the Johns Hopkins University as James Speyer Visiting Professor of Mathematics from March 2 to May 8, 1931, on "Topological Questions in Differential Geometry."

Professor Leonida Tonelli, of the University of Bologna, has been appointed to the professorship of infinitesimal analysis at the University of Pisa. He will be succeeded at Bologna by Professor Giuseppe Vitali, of the University of Padova.

Dr. R. Caccioppoli has been named professor of algebra in the University of Padua.

Dr. G. Mignosi has been named professor of algebra in the University of Cagliari.

Professor Albert Einstein will be in residence at Oxford University during the summer of 1931; he will deliver the Rhodes memorial lecture for the year 1930–31.

Dr. R. A. Fisher, Chief Statistician of Rothamsted Agricultural Experimental Station, Harpenden, England, will be in residence on the staff at the Iowa State College during the first half of the summer session, June 16 to July 24 and later will visit a number of colleges, universities, and experiment stations throughout the country. In connection with Doctor Fisher's visit, Iowa State College announces a special group of summer session courses covering the theory and application of statistics.

At Trinity College, Cambridge, A. S. Besicovitch and L. Wittgenstein have been elected to fellowships.

The following seventy-six doctorates with mathematics or mathematical physics as major subject were conferred during 1930 in universities in the United States and Canada; the major subject is mathematics unless otherwise specified. The university, month in which the degree was conferred, minor subject (other than mathematics), and title of dissertation are given in each case if available.

Virgil Adkinsson, Pennsylvania, June, *Cyclically connected continuous curves whose complementary domain boundaries are homeomorphic, preserving branch points.*

R. P. Agnew, Cornell, June, minor in physics, *The behavior of bounds and oscillations of sequences of functions under regular transformations.*

Rose L. Anderson, Bryn Mawr, June, minor in physics, *A problem in the simultaneous reduction of two quadratic forms in infinitely many variables.*

C. A. Andree, Wisconsin, June, major in electrical engineering physics, minor in mathematics, *An impact strain recorder.*

J. V. Atanasoff, Wisconsin, June, major in mathematical physics, minor in mathematics, *The dielectric constant of helium.*

R. F. Bacher, Michigan, June, *The Zeeman effect of hyperfine structure.*

E. B. Baker, Michigan, June, *The application of the Fermi statistics to the calculation of the potential distribution of positive ions.*

C. F. Bowles, Chicago, August, *Integral surfaces of pairs of differential equations of the third order.*

M. G. Boyce, Chicago, March, *An envelope theorem and necessary conditions for a problem of Mayer with variable end-points.*

Leonard Bristow, Illinois, May, minor in physics. *Expansion theory associated with linear differential equations and their regular singular points.*

L. H. Bunyan, Wisconsin, June, *A transformation of a certain integral equation and a theorem concerning an integro-differential equation.*

J. H. Bushey, Michigan, June, *Asymptotic expressions for a certain class of definite integrals.*

Hung Chi Chang, Michigan, June, *Transformation of linear partial differential equations.*

Elizabeth M. Cooper, Illinois, June, minor in physics, *Perspective elliptic curves.*

J. J. Corliss, Michigan, June, *On the unsymmetric top.*

A. E. Currier, Harvard, June, *The problem of the calculus of variations in m-space with end points variable on two manifolds.*

E. H. Cutler, Harvard, June, *Some properties of general subspaces of a Riemann space.*

M. N. Davis, Wisconsin, June, major in physics, minor in mathematics, *A study and partial explanation of the emission of secondary electrons from cobalt.*

E. H. Dixon, Wisconsin, June, major in physics, minor in mathematics, *The photoelectric and thermionic properties of rhodium.*

L. H. Donnell, Michigan, June, *Some examples of mechanical wave transmission.*

W. L. Duren, Jr., Chicago, August, *The development of sufficient conditions in the calculus of variations.*

W. H. Durfee, Cornell, February, minor in physics, *Summation factors which are powers of a complex variable.*

L. A. Dye, Cornell, June, minor in physics, *Involutorial transformation in S_3 of order n with an $(n-1)$-fold line.*

G. L. Edgett, Illinois, August, minor in astronomy, *Frequency distributions with given statistics which are not all moments.*

C. M. Erikson, Michigan, June, *Systems of linear difference equations with constant coefficients*

O. J. Farrell, Harvard, June, (1), *On the expansion of harmonic functions in*

terms of harmonic polynomials; (2), *On approximation to an arbitrary function of a complex variable by polynomials.*

E. J. Finan, Ohio State, August, *Determination of the domains of integrity of the complete rational matric algebra of order four.*

W. W. Flexner, Princeton, June, *On topological manifolds.*

H. L. Garabedian, Princeton, June, *On the relation between certain methods of summability.*

B. P. Gill, Columbia, July, *An analogue for analytic functions of the Thue-Siegel theorem.*

Beatrice L. Hagen, Chicago, August, *Quintuples of three dimensional varieties in a four-dimensional linear space.*

Frances Harshbarger, Illinois, May, minor in physics, *The geometric configuration defined by a special algebraic relation of genus four.*

G. A. Hedlund, Harvard, June, (1), *Geodesics on a two-dimensional Riemannian manifold with periodic coefficients*; (2) *Poincaré's rotation number and Morse's type number.*

V. A. Hoyle, Princeton, June, *Some problems in conformal geometry.*

H. K. Hughes, Michigan, June, *On the analytical extension of functions defined by factorial series.*

Aline Huke, Chicago, March, *An historical and critical study of the fundamental lemma in the calculus of variations.*

F. C. Jonah, Brown, October, *The Green's matrix and expansion problem for systems of integro-differential equations.*

T. H. Kiang, Harvard, June, *Existence of critical points of harmonic functions of three variables.*

Edna E. Kramer, Columbia, June, *On the Laguerre group and allied topics.*

C. C. Krieger, Toronto, June, minor in the theory of relativity, (1), *On the summability of trigonometrical series with localized properties*; (2) *On Fourier constants and convergence factors of double Fourier series.*

H. I. Lane, Cornell, September, *The separation of the projective plane by the lines joining six points.*

Voris Latshaw, Indiana, June, minor in physics, *The algebra of self-adjoint boundary value problems.*

D. H. Lehmer, Brown, June, *An extended theory of Lucas' functions.*

C. N. Liu, Harvard, June, *Contributions to the restricted problem of three bodies.*

Shu Ting Liu, Michigan, June, *Theory of periodic orbits for asteroids of integral types.*

W. H. McEwen, Minnesota, July, minor in physics, *Problems of closest approximation connected with the solution of linear differential equations.*

E. J. McShane, Chicago, June, *Semi-continuity in the calculus of variations, and absolute minima for isoperimetric problems.*

C. W. Mendel, Chicago, June, *Contributions to the projective differential geometry of hyperspace.*

C. A. Messick, Chicago, August, *Symmetric functions of infinitely many elements.*

E. W. Miller, Michigan, June, *Concerning subsets of a continuous curve which lie on an arc of the continuous curve.*

Ethel I. Moody, Cornell, June, minor in philosophy, *A Cremona group of order thirty-two of cubic transformations in three dimensional space.*

G. C. Munro, Michigan, June, *Systems of linear partial differential equations with constant coefficients.*

Alexander Oppenheim, Chicago, June, *The minima of indefinite quaternary quadratic forms.*

T. S. Peterson, Ohio State, June, *The invariant theory of functional forms under the group of linear functional transformations of the third kind.*

O. H. Rechard, Wisconsin, June, minor in philosophy, *The expansion problem associated with a class of ordinary differential boundary-value problems.*

D. P. Richardson, Chicago, August, *Quadratic Cremona transformations of five-dimensional space.*

D. A. F. Robinson, Chicago, June, *Fourier expansions of pseudo doubly periodic functions with applications.*

Ragner Rollefson, Wisconsin, June, major in physics, minor in mathematics, *The continuous and band spectrum of mercury.*

L. R. Salvosa, Michigan, June, *Generalizations of the normal curve of error.*

H. W. Sibert, Cincinnati, June, *Moderately thick plates with plane faces.*

R. G. Smith, Kansas, June, minor in physics, *A canonical form for the differential equations of curves in n-dimensional space.*

Elizabeth T. Stafford, Wisconsin, June, *Matrices conjugate to a given matrix with respect to its minimum equation.*

G. W. Starcher, Illinois, May, minor in physics, *On identities arising from solutions of q-difference equations and some interpretations in number theory.*

H. E. Stetson, Iowa, February, minor in physics, *Double function space.*

C. W. Strom, Illinois, June, minor in physics, *On complete systems under certain finite groups.*

Mildred E. Taylor, Illinois, August, minor in physics, *A determination of the types of planar Cremona transformations with not more than 9 F-points.*

H. P. Thielman, Ohio State, June, (1), *On new integral addition theorems for Bessel functions and series of the hypergeometric type*; (2), *The application of functional operations to a class of integral equations occurring in physics.*

W. R. Thompson, Yale, June, *The possible forms of discriminants of algebraic fields.*

R. S. Underwood, Chicago, June, *On universal quadratic null forms in five variables.*

Charles Wexler, Harvard, June, *On the theory of quadratic fields.*

J. H. C. Whitehead, Princeton, June, *The representation of projective space.*

F. L. Wren, Chicago, March, *A new theory of parametric problems in the calculus of variations.*

H. M. Yarbrough, Indiana, June, minor in astronomy, *Linear cyclic groups with given fundamental region.*

R. C. Yates, Johns Hopkins, June, *The small vibrations of certain mechanical systems.*

J. Yerushalmy, Johns Hopkins, June, *The construction of equianharmonic cubics.*

L. A. Young, Michigan, June, *The Wentzler-Brioullin-Kramers approximate solution of the Schrödinger wave equation.*

Dr. Nola L. Anderson has been appointed associate professor and acting chairman of the department of mathematics at Sophie Newcomb College.

Professor G. B. Drummond, formerly at the Mississippi Agricultural and Mechanical College, has been appointed an assistant professor of mathematics at the Oklahoma Agricultural and Mechanical College.

Dr. Wolfgang Pauli, of the Zurich Technical School, and Dr. Arnold Sommerfeld, of the University of Munich, have been appointed lecturers in theoretical physics at the University of Michigan for the summer session of 1931.

Dr. N. B. MacLean, formerly professor of mathematics at the University of Winnipeg, has been appointed professor of Mathematics at McGill University, as successor to Professor D. A. Murray, retired.

The following appointments to instructorships in mathematics are announced:
Iowa State College, J. C. Hempstead;
University of Michigan, Dr. J. D. Elder.

Professor P. Gazzaniga, of the University of Padova, died October, 18, 1930, at the age of seventy-seven.

Dr. E. Stübler, professor of descriptive geometry, financial mathematics, and insurance mathematics at the Charlottenburg Technical School, died in December, 1930, at the age of fifty-eight.

Dr. Frantisek Wald, professor of physical chemistry at the Czech Technical School at Prague, died October 19, 1931.

Professor R. A. Sheets, of Denison University, died December 3, 1930.

Professor H. E. Trefethen, a teacher of mathematics and astronomy in Colby College for nearly twenty years, died on November 3, 1930.

Professor J. Ricard, S. J., formerly professor of mathematics and director of the observatory at the University of Santa Clara, has died, at the age of eighty.

ABSTRACTS OF PAPERS

SUBMITTED FOR PRESENTATION TO THIS SOCIETY

The following papers have been submitted to the Secretary and the Associate Secretaries of the Society for presentation at meetings of the Society. They are numbered serially throughout this volume. Cross-references to them in the reports of the meetings will give the number of this volume, the number of this issue, and the serial number of the abstract.

114. Dr. W. Cauer: *Electric networks and bounded functions.*

The problem of designing communication networks with prescribed external characteristics corresponds essentially to the following: Given a complex variable λ, three positive quadratic forms of n variables with matrices L, R, D; to find the conditions that a given minor matrix of m-th order of functions is a chief minor of some A^{-1} where $A=\lambda L+R+\lambda^{-1}D$; then to find one or all sets of the original L, R, D which produce the given minor. A^{-1} is regular in the whole right half-plane, real for real λ, and the real part of A^{-1} belongs to a positive definite quadratic form whenever λ is a point of the right half-plane. The converse, that under these conditions one may construct positive definite L, R, D producing the given minor, seems to be true in general. This paper proves this theorem for: (1) $m=1$ (a) under restriction of degree of the driving point impedence corresponding to the R. M. Foster case (Bell System Technical Journal, 1924, p. 651), but $n=2$ or 3, (b) n infinite. (2) $m=2$, minor symmetric with respect to the secondary diagonal (symmetrical four-terminal network). Graphically given minors are considered. (Received December 20, 1930.)

115. Dr. W. Seidel (National Research Fellow): *On the approximation of continuous functions by linear combinations of continuous functions.*

The author presents a new proof of the following theorem of F. Riesz: *Let $\phi_1(x)$, $\phi_2(x)$, $\cdots$, $\phi_n(x)$, $\cdots$ be a sequence of arbitrary, linearly independent functions, defined and continuous in the interval $a\leqq x\leqq b$. A necessary and sufficient condition that linear combinations of these functions uniformly approximate every function $f(x)$, defined and continuous in $a\leqq x\leqq b$, is that there shall exist no function $\alpha(x)$, of bounded variation, satisfying the system of integral equations $\int_a^b\phi_i(x)d\alpha(x)=0$, $i=1, 2, \cdots$, except if $\alpha(x)$ is constant in the interval $a\leqq x\leqq b$ save perhaps for a denumerable set of values of x different from a and b.* Consider an $(n+1)$-dimensional euclidean space $(x_0, x_1, \cdots, x_n)$ and define the two curves $\Gamma_n^+: x_0=f(x)$, $x_1=\phi_1(x)$, $\cdots$, $x_n=\phi_n(x)$ and $\Gamma_n^-: x_0=-f(x)$, $x_1=-\phi_1(x)$, $\cdots$, $x_n=-\phi_n(x)$. Let $K_n(f, \phi_1, \cdots, \phi_n)$ be the smallest convex body containing Γ_n^+ and Γ_n^-. The origin O surely lies in $K_n(f, \phi_1, \cdots, \phi_n)$. Let the line joining O with the point $(1, 0, \cdots, 0)$ cut $K_n(f, \phi_1, \cdots, \phi_n)$ in a point A_n. The proof of the theorem is based on the following lemma: There

will exist linear combinations of the functions $\{\phi_i(x)\}$ converging uniformly toward $f(x)$ in the interval $a \leqq x \leqq b$ if, and only if, $\lim_{n\to\infty} \overline{OA}_n = 0$. The lemma is obtained by altogether elementary geometric considerations, and the theorem of Riesz follows almost immediately from it. (Received February 7, 1931.)

116. Dr. W. Cauer: *An interpolation problem of functions with positive real part.*

The design of symmetrical electric wave filters which have nearly zero "attenuation" in certain frequency intervals corresponds essentially to the solution of the following interpolation problem: $z_1 = f_1(\lambda)$ and $z_2 = f_2(\lambda)$ are rational functions of λ which are regular in the right half plane with positive real part and real on the real axis; find z_1 and z_2 so that $\log [(1+z_1)(1+z_2)/(z_2-z_1)]$ is approximately zero in certain given intervals of the positive imaginary axis and ∞ in the complementary intervals. For this it is necessary and sufficient that z_1 and z_2 be (nearly) pure imaginary on the imaginary axis, $z_1 z_2$ be approximately 1 in the first named intervals, and z_1/z_2 be approximately 1 in the complementary intervals. Both these reduced interpolation problems are independent and of the same kind. Several solutions are given. One shows the limiting case where the degree of the functions z_1 and z_2 goes to infinity and the approximation becomes ideal. Especially valuable is an approximation in the sense of Tchebycheff which is related to the transformation of elliptic integrals. The Tchebycheff parameters are calculated explicitly for all important types of filters. (Received February 7, 1931.)

117. Professor J. L. Walsh: *On interpolation and approximation by rational functions with preassigned poles.*

If C is an arbitrary limited *closed* Jordan region containing no limit point of the given set α_{in}, $i = 1, 2, \cdots, n$; $n = 1, 2, \cdots$, and if $f(z)$ is an arbitrary function analytic interior to C, and continuous in C, then there exist rational functions $r_n(z) = (a_{0n}z^n + a_{1n}z^{n-1} + \cdots + a_{nn})/[(z-\alpha_{1n})(z-\alpha_{2n}) \cdots (z-\alpha_{nn})]$, $n = 0, 1, 2, \cdots$, approaching $f(z)$ uniformly in C. In particular, if C is the circle $|z| \leqq 1$, if the points α_{in} have no limit point interior to $|z| = A > 1$, and if $f(z)$ is analytic for $|z| < T > 1$, then the sequence of functions $r_n(z)$ of *best* approximation to $f(z)$ on C (this sequence exists and is unique) converges to $f(z)$ uniformly for $|z| < R < (A^2T + T + 2A)/(2AT + A^2 + 1)$. This result holds whether *best approximation* is measured in the sense of Tchebycheff, in the sense of least weighted pth powers ($p > 1$) on the circumference, or in the sense of least weighted pth powers ($p > 1$) over the area C. There are analogous results for sequences $r_n(z)$ obtained by interpolation. (Received February 2, 1931.)

118. Dr. L. S. Kennison: *Reflections in function space.*

J. Delsarte has announced the following theorem: The necessary and sufficient condition that a kernel be a kernel of rotation (as defined by Delsarte) is that it be the resolvent of a skew-symmetric kernel with parameter 1/2. An example is given which shows that the condition is not necessary. The theorem is corrected and generalized. (Received January 19, 1931.)

119. Professor J. F. Ritt: *Quotients of differential forms.*

Let $A(u)$ and $B(u)$ be two polynomials in an arbitrary analytic function u, and its derivatives, with analytic coefficients. Let $y=A(u)/B(u)$. The question arises as to how y is to be determined if u makes both A and B vanish. What one may do, following the theory of differential equations recently developed by the author in the Transactions of this Society, is to take all functions y which, with the given u, give a solution of $By-A=0$, belonging to the *general solution* of that equation. This point of view leads to specific results of some interest. Let $F(u)$ and $G(u)$ be two rational combinations of u and its derivatives, with analytic coefficients. It is proved that there exists a third such rational combination, $H(u)$, such that H is rational in F, G and their derivatives, while F and G are each rational in H and its derivatives. This is analogous to an algebraic theorem of Lüroth. (Received February 10, 1931.)

120. Mr. W. O. Pennell: *Representation by Fourier series of functions whose successive intervals lie in separate planes differing by a phase angle.*

Suppose $\chi(x)$ is a function defined in the interval $0<x<a$ and satisfying the necessary conditions for representation in this interval by a classical Fourier series. This paper describes a quasi-Fourier series $S(x)$ in space representing the following function: $S(x)=[b^n\chi(x-na)]_{n\psi}$ in the intervals $na<x<(n+1)a$, where a, b, and ψ are any real constants and n takes the values 0, ±1, ±2, ±3, $\cdots$ corresponding to the above intervals and the subponent notation $n\psi$ denotes the angles which the planes passing through $\chi(x-na)$ and the X axis for each interval make with the XY plane. An illustrative example is given with models showing the approximation curves twisting through space about the final form of the function in question. (Received January 24, 1931.)

121. Mr. L. E. Widmark: *On repeat operations.*

The notation $R(f(x))$ expresses a continued operation of a function upon itself. It is shown that this conception leads to a short and perspicuous notation for many at present unwieldy expressions. It also leads to a convergency solution of numerical equations which is believed new and of practical value. (Received February 6, 1931.)

122. Dr. T. C. Benton: *On continuous curves which are homogeneous except for a countable infinity of points.*

The problem of extending the results of the author's papers *On continuous curves except for a finite number of points* (Fundamental Mathematicae, vols. 13, 15) to obtain a classification of such curves with a countable infinity of non-homogeneous points gives the result that in any bounded curve of the required type the set of non-homogeneous points has a derived set which consists of a finite number (n) of points. If this number (n) is greater than one, all such curves can be constructed by taking a curve with n non-homogeneous points and replacing each arc of it by a simple type of curve homogeneous except for a countable infinity of points. If $n=1$ the problem is much more complicated. Only certain cases of it have been worked out. (Received February 7, 1931.)

123. Dr. A. A. Albert: *A classification of all pure Riemann matrices of genus $p \leqq 8$.*

Pure Riemann matrices of genus $p=2, 3$ have been classified by several authors. Their classifications are all in terms of the invariant indices of the matrices, not the far more fundamental structure of the associated multiplication algebras which also gives these indices. As a consequence the methods used failed of extension to the case $p=4$. In the present paper the author applies his recent theory of the structure of a pure Riemann matrix with a given algebra of multiplications to give a complete classification of pure Riemann matrices of genus $p \leqq 8$. In fact, a classification is given for $p=8$, $p=rp_1$, where $r=1, 2, 4$, and p_1 is unity or an odd prime. Moreover a study is made of all Riemann matrices whose multiplication algebras are those algebras occurring in the above special cases, general existence theorems being proved not merely for the case of ordinary Riemann matrices but for Riemann matrices over any real algebraic field. These algebras are algebraic fields, and three types of generalized quaternion division algebras. (Received January 28, 1931.)

124. Professor R. M. Garver: *On the Bring-Jerrard quintic.*

Most of the proofs that the general quintic equation can be reduced to the form $x^5+ax+b=0$ make no attempt to determine a and b explicitly, while those that do are very long. (See, for example, Rahts, Mathematische Annalen, vol. 28 (1886), pp. 34–60.) The purpose of the present paper is to devise a transformation, or series of transformations, which will allow a and b in the transformed equation to be computed fairly conveniently. (Received February 6, 1931.)

125. Professor R. M. Garver: *Tschirnhaus transformations and the real roots of an equation.*

Considerable use has been made of Tschirnhaus transformations in the determination of the number of real roots of equations. (See Fricke, *Algebra*, vol. 1, pp. 189 ff. and pp. 217 ff.) These applications are not, however, of an elementary nature. This paper shows how to obtain the desired criteria for quartic and quintic equations with the aid of only simple transformations and symmetric functions. (Received February 6, 1931.)

126. Dr. E. H. Cutler: *On the curvatures of a curve in Riemann space.*

An attempt is made to extend to Riemann spaces three properties of a curve in ordinary space, namely (1) the formula for the principal part of the infinitesimal distance of a point of the curve from the k-dimensional osculating geodesic space at a nearby point, (2) the equality of the first $(k-1)$ curvatures of the curve and those of its projection on the k-dimensional osculating geodesic space, and (3) the fact that if the kth curvature vanishes identically the curve lies in the k-dimensional osculating geodesic space. None of these properties holds in general, but all hold if the osculating geodesic space in question is totally geodesic, and (1) and (2) hold for the first and second curvatures. (Received February 4, 1931.)

127. Professor J. W. Lasley, Jr.: *Penosculating conics of a plane curve.*

This paper presents the results of studying certain penosculating conics associated with a given plane curve at an ordinary point of it. Constructions for these conics are obtained, and additional properties and relationships pointed out. The auxilliary parabola of Transon and the ellipse of minimum eccentricity of Wilczynski are objects of especial study. Certain results already known are extended. An involution of conics comprising the entire system of penosculating conics is introduced, generalizing results due to Wilczynski. (Received January 26, 1931.)

128. Dr. S. B. Littauer (National Research Fellow): *On convex arcs and surfaces.*

The area of a convex surface in three-space admits an intrinsic definition analogous to that for arc length, that is, as the upper bound of the areas of all the convex polyhedra inscribed in the surface. Since for two convex regions R and R' such that R is a subset of R', the boundary of R has a smaller area than that of R', the definition is equivalent to Lebesgue's within its limits of application. Arc length, it is well known, is a lower semicontinuous functional. Restricted to convex arcs, length is continuous; and with respect to the whole system of convex arcs of length greater than 0 any finite cube is compact. (Received February 2, 1931.)

129. Professor R. G. Putnam: *On spaces V_ω.*

In this paper complete spaces V_ω are defined and two theorems similar to Theorems 1 and 2 of a paper by Kuratowski (Fundamenta Mathematicae, vol. 15, p. 301) on complete metric spaces are shown to be valid in complete spaces V_ω. Theorems 3 and 4 of the same paper are extended to certain spaces V_ω. Four theorems shown by the author (this Bulletin, 1930, p. 127) to be true in spaces E are extended to spaces V_ω. (Received January 30, 1931.)

130. Mr. Hassler Whitney: *On homeomorphic graphs and the connectivity of graphs.*

Definitions will be found in a note, *Non-separable and planar graphs*, in the Proceedings of the National Academy of Sciences, February, 1931. Let G and G' be two graphs containing no 1- or 2-circuits, and let there be a (1, 1) correspondence between their arcs. Then G and G' are homeomorphic if (1) neither of the graphs is of the form ab, ac, ad. and two arcs with a common vertex in one correspond to two arcs with a common vertex in the other, or (2) the graphs are triply connected, and circuits correspond to circuits, or (3) the graphs are triply connected, and subgraphs of nullity 1 correspond to subgraphs of nullity 1. A graph is n-tuply connected if it cannot be disconnected by dropping out fewer than n vertices and their arcs. If a graph is n-tuply connected, any two of its vertices are joined by n distinct chains, and conversely. Applications to dual graphs are given: dual graphs have the same connectivity; a triply connected planar graph has a unique dual. (Received February 6, 1931.)

131. Professor Edward Kasner: *Invariance under contact transformation.*

The simplest type of differential equation invariant under all contact transformations is

$$y''' = ay''^3 + by''^2 + cy'' + d.$$

This plays the same rôle as the familiar type $y'' = Ay'^3 + By'^2 + Cy' + D$, which is of first *rank* in the author's classification for the group of all point transformations given in the American Journal of Mathematics, 1906. Besides families of ∞^3 curves, we may construct families of $2\infty^2$ curves which are contact-invariant. For example, all families obtained by applying contact transformation to the ∞^2 straight lines and the ∞^2 point-unions sonstitute such a type. We may also construct families of $\infty^2 + 3\infty^1$ curves. Such invariant family types may be characterized geometrically by a generalization of Blaschke's hexagonal web (which itself is of course only topologically invariant). If we are given $3\infty^2$ curves (that is, such that for any lineal element there are three curves) we may construct certain related families of *anharmonic-curvature trajectories.* This gives in all ∞^3 curves, and forms a special case of the cubic type described in the first sentence. This sub-type is contact-invariant. (Received January 31, 1931.)

132. Dr. E. H. Cutler: *On the Frenet formulas for a general subspace of a Riemann space.*

Starting with a general subspace of a Riemann space, it is shown that, by a proper choice of systems of normals and by a corresponding definition of complete convariant differentiation, formulas analogous to the Frenet formulas for a curve will be obtained. The quantities occurring there will satisfy the Gauss, Codazzi, Ricci equations, and will transform as tensors under a change of coordinate system in the subspace or under a rotation of the reference normals of any system among themselves. An extension of Meusnier's theorem is developed, and certain theorems about curves are extended to the general subspace. (Received February 4, 1931.)

133. Dr. A. A. Albert: *The integers represented by sets of positive ternary quadratic forms.*

Almost no general theorems are known on explicitly what integers are represented by positive ternary quadratic forms. In fact, as L. E. Dickson has indicated (Annals of Mathematics, vol. 28), most of these forms are *irregular*. The author studies here the problem of determining all integers represented by the set $\Sigma(d)$ of all positive classical ternaries of the same determinant d. The problem is completely solved, and it is shown that if we write $d = \gamma^2\delta$, where δ has no square factor, then $\Sigma(d)$ represents all positive integers a not of the form $a = \lambda^2\mu$, where μ has no square factor, $\mu = \alpha\delta$, α is prime to d, $\alpha \equiv 7 \pmod 8$, and the Jacobi symbol $(f|\alpha) = 1$ for every factor f of d. This result is easily shown to imply that every set $\Sigma(d)$ is regular in the Dickson sense and that no such set represents all positive integers. Also every set $\Sigma(n, d)$, $n > 3$, of all positive classical n-ary quadratic forms of determinant d represents all positive integers. (Received January 28, 1931.)

134. Dr. Frances Harshbarger: *The geometric configuration defined by a special algebraic relation of genus* 4.

This paper discusses the curve defined by the special form $l_1^3\tau_1^2\tau_2+l_1^2l_2\tau_2^3+l_1l_2^2\tau_1^3-l_2^3\tau_1\tau_2^2=0$, where l_1/l_2 and τ_1/τ_2 define the generators of a quadric surface. This form was first investigated by Gordan (Mathematische Annalen, vol. 13 (1878)) in connection with the solution of the quintic equation. (Received February 14, 1931.)

135. Professors C. L. E. Moore and Philip Franklin: *Geodesics of Pfaffians.*

In a Pfaffian, there is a class of curves whose principal normals are normal to the Pfaffian. These have been called "geodesics," although their dimensionality is one lower than that of pairs of points in the Pfaffian. We here study the lines of shortest length joining pairs of points, obtaining their differential equations, their finite form for some simple examples, and some of their local properties. The earlier class are only curves of shortest length for integrable Pfaffians. (Received February 11, 1931.)

136. Professors C. L. E. Moore and Philip Franklin: *Pfaffians in parametric form.*

In this paper we show that the method of parametric representation for surfaces, which has been assumed by some writers to be applicable to Pfaffians, only applies to certain very restricted classes of Pfaffians. These classes are determined, as well as those types of Pfaffian in three-space for which the arc element can be represented in terms of two parameters. (Received February 11, 1931.)

137. Professor Philip Franklin: *Regions of positive and negative curvature on closed surfaces.*

The question as to topological restrictions on regions of positive and negative curvature has been raised by J. Douglas (this Bulletin, vol. 36, p. 798). In this note we show by examples that the conjectured properties do not hold for general closed surfaces. If, however, we impose the restriction that the points on a surface of genus one at which the curvature is zero form simple curves, and include no umbilics or points at which the normal section to the surface has a point of inflection, the regions in question are not shrinkable to a point. (Received February 11, 1931.)

138. Professor Philip Franklin: *Two functional equations with integral arguments.*

E. T. Bell (American Mathematical Monthly, vol. 37, p. 484) has formulated two functional equations with integral arguments whose general solution is of importance in a question on types of arithmetic. We here solve them and show that the general solution of the first equation involves a single function of the variable, while that of the second involves an enumerable number of such functions. (Received February 11, 1931.)

139. Professor Philip Franklin: *The geometric interpretation of some elementary formulas of analytic geometry.*

The condition that three points lie on a line is an expression of the fact that the area of the triangle with these points as vertices vanishes. The connection of the condition that three lines meet in a point with the vanishing of the triangle having these lines as sides is not so direct, and has not been explicitly brought out. In this note we show that when the equations are suitably normalized, the condition admits of geometric interpretation. The generalization to three and n dimensions is indicated. (Received February 11, 1931.)

140. Miss Ruth W. Stokes: *A geometric theory of solution of linear inequalities.*

The author developes a theory of systems $\xi^i \geqq 0$, where the ξ's are linear homogeneous expressions in n unknowns. Each condition is represented in n-dimensional euclidean space by the point which has for coordinates the known coefficients of the corresponding form ξ. A solution is accordingly an oriented $(n-1)$-flat which does not separate any pair of the points representing the given conditions. The paper determines the general solution of the following problem: find a solution $\xi \geqq 0$ which contains all points in a subset A of the given representative points and which contains no point in a second subset B. A general solution is thus obtained for any system composed of a finite number of conditions from each of the three types (1) $\xi \geqq 0$, (2) $\xi > 0$, (3) $\xi = 0$. Such a system includes as special cases the system composed solely of conditions of type (1) studied by Minkowski (*Geometrie der Zahlen*, pp. 40–45); that composed solely of conditions of type (2) studied by Dines (Annals of Mathematics, vol. 20, pp. 191–199); and that composed solely of conditions of type (3), which is a system of linear homogeneous equations. The method is geometric throughout. (Received March 4, 1931.)

141. Mr. C. C. Torrance: *On plane Cremona triadic characteristics.*

In this paper is developed a new procedure for obtaining the characteristics of all plane Cremona transformations. This procedure is perfectly regular and can be simply and explicitly formulated. Furthermore it develops the characteristics in such a way that all the characteristics obtained by any given number of steps are included in one single general formula. This formula includes only geometric characteristics. The latter part of the paper is devoted to the beginning of an exhaustive investigation of the ways in which the parameters introduced may be evaluated so that triadic characteristics result. Twelve infinite sequences of two-parameter triadics are obtained and their conjugates determined. All of these triadics have for one of their groups a single point of highest multiplicity. (Received March 4, 1931.)

142. Professor C. C. MacDuffee: *A method for determining the canonical basis of an ideal in an algebraic field.*

While the existence of a basis for every ideal in an algebraic field is easily proved, an effective method for calculating it has been developed only in the

case of quadratic ideals (M. Cipolla, Atti Academie Gionia Scientica Nationale in Catonia (5), vol. 10 (1917), No. 20), and the method used is too complicated to invite attempts to generalize it. It is here shown that a basis for the ideal of $(a_1, a_2, \cdots, a_k)$ can easily be obtained in matric form as the greatest common right divisor of certain integral matrices obtained directly from the a's. The process is entirely practicable. (Received January 28, 1931.)

143. Dr. E. J. McShane, National Research Fellow: *On the necessary condition of Weierstrass in the multiple integral problem of the calculus of variations.*

The principal result arrived at in this paper is the following: Given a class K of functions $z(x, \cdots, w)$ for which the point $(x, \cdots, w)$ lies in a set D of n-space and $(z, x, \cdots, w)$ in a set A of $n+1$-space, and an integral $F=\int_D f(x, \cdots, w, z, z_x, \cdots, z_w)dx \cdots dw$. If the function $\zeta(x)$ minimizes F in K, and L is the set of points $(x, \cdots, w)$ interior to D for which $(\zeta(x, \cdots, w), x, \cdots, w)$ is interior to A and of indifference with respect to K and A, and near which ζ satisfies a Lipschitz condition, then for almost all points of L the Weierstrass function $E(x, \cdots, w, \zeta, \zeta_x, \cdots, \zeta_w, z_x, \cdots z_w)$ is non-negative for all sets of numbers $z_x, \cdots, z_w$. If f satisfies a certain supplementary condition, we can remove the restriction that ζ satisfy a Lipschitz condition near $(x, \cdots, w)$. (Received January 27, 1931.)

144. Professor Ernest P. Lane: *Conjugate nets and the lines of curvature.*

The purpose of this paper is twofold; first, to make some contributions to the profective differential geometry of an arbitrary conjugate net on a general analytic surface in ordinary space; and second, to connect this geometry with the metric differential geometry of the lines of curvature. Two quadrics called *conjugate osculating quadrics* are associated with each point of a curve on the surface sustaining a given conjugate net. These are analogous to the *asymptotic osculating quadrics* of Bompiani and Klobouček, the difference being that three consecutive tangents of one family of the given conjugate net are constructed at a point of the curve to determine a quadric, instead of three consecutive asymptotic tangents. The transition to metric geometry is affected by a transformation which makes it possible to study metrically for the lines of curvature all configurations defined in the projective theory for a general conjugate net in ordinary space. (Received February 4, 1931.)

145. Mr. E. S. Wilks: *On the distribution of statistics in samples from a normal universe of two variates with matched sampling of one variable.*

A type of sampling frequently used in statistical investigations may be formulated abstractly by considering a sample of s items drawn from a normally distributed universe of two variables x, y in such a way that the distribution of the x's is made identical with a given distribution $D(x)$, independently of the y's. This paper deals with the derivation of the distribution functions of the

important statistics pertaining to the y's for sampling under such restrictions. The method used may be described in a general way as an extension of Romanovsky's integral equation method of finding distributions of statistics from normal universes for strictly random sampling. Expressions were also found for the moments, expected values and standard errors of these statistics, some of which were obtained in the form of confluent hypergeometric series which were transformed into power series in $1/s$ by a relation due to Kummer, thus enabling one to make convenient approximations. (Received February 20, 1931.)

146. Dr. J. J. Gergen: *Note on a theorem of Bôcher and Koebe.*

In this paper the following theorem is established:

Theorem I. If $v(x, y)$ is positive and harmonic in a plane region R, if $u(x, y)$ is continuous with its first partial derivatives in R, and if, for every circle C contained in R, $\int_C v\partial u/\partial n\, ds = \int_C u\partial v/\partial n\, ds$, where n is the exterior normal to C, then u is harmonic in R. Taking $v=1$ here, a well-known theorem, discovered independently by Bôcher and Koebe, is obtained. The analog of Theorem I holds in space. (Received February 24, 1931.)

147. Dr. J. Yerushalmy, National Research Fellow: *On the representative space S_9 of the plane cubics.*

The plane cubics are mapped on the points of an S_9. Cubics with a given singularity on the points of a spread in S_9. The orders of the various spreads are determined as well as their multiplicities with respect to each other, and the linear spaces by which they are ruled. A more detailed study is made of the surface F_2^9 of cubics degenerating into a triple line determining the curves on it, its tangentsplanes and hyperplanes, its projection on S_3, and its map on the plane. Most of the results have been extended to the general representative space $S_N (N=n(n+3)/2)$ of the curves of order n in the plane. (Received March 1, 1931.)

148. Professor H. T. Davies: *Properties of the operator $z^{-v} \log z$, where $z=d/dx$.*

The operator discussed by this paper is derived from the fractional operator z^{-v} by differentiation with respect to v. Its inverse is obtained by means of a proper integration from 0 to ∞ and various generalizations are suggested. The paper shows that the generalized Leibnitz formula for the expansion of the operation $F(x, d/dx)uv$ holds when $F(x, z)$ is the logarithmic operator and that in consequence the Bourlet operational product is also valid when logarithmic terms are included in either factor. Commutation rules for the operator are derived and application is finally made to the solution of several types of integral equations. (Received March 2, 1931.)

149. Professor H. S. Wall: *Convergence criteria for continued fractions.*

A certain transformation $b=[a]$ takes the continued fraction $F(a, z)=1/a_1 z+1/a_2+1/a_3 z+\cdots$, into $F(b, z)$, (a_i, b_i real and not zero). If $\sum |b_i|$ converges $F(a, z)$ converges if and only if $\lim_n |g_n^a| = \infty$, where $g_n^a+a_2+a_4+\cdots$

$+a_{2n}$, $(g_0^a=0)$. Let $c=[b]$ take $F(b, z)$ into $F(c, z)$. Then if $\sum|c_i|$ converges $F(a, z)$ converges if and only if (i) $\sum c_{2i+1}=0$ or (ii) $\sum c_{2i+1}\neq 0$, $\sum b_{2i+1}=0$. When $F(a, z)$ converges the limit is a meromorphic function of z. Explicit formulas for the b_i, c_i are

$$b_{2i+1}=b_1{}^2a_{2i}/(1-b_1g_1{}^a)(1-b_1g_i{}^a{}_{-1}),\ b_{2i+2}=a_{2i+1}(1-b_1g_i{}^a)^2/b_1{}^2,$$
$$c_{2i+1}=c_1{}^2a_{2i-1}(1-b_1g_i{}^2{}_{-1})^2/b_1{}^2(1-c_1g_i{}^b)(1-g_i{}^b{}_{-1}),$$
$$c_{2i+2}=b_1{}^2a_{2i}(1-c_1g_i{}^b)^2/c_1{}^2(1-b_1g_i{}^a)(1-b_1g_i{}^a{}_{-1}).$$

(Received March 3, 1931.)

150. Professor John Eiesland: *On the ruled $V_4^{(4)}$ in S_5 associated with a Schlafli hexad of 3-flats.*

The present paper gives an account of the singular loci of a ruled $V_4^{(4)}$ in S_5, the general equation of which has been deduced by C. R. Rupp, (Transactions of this Society, vol. 31, p. 587). By a projective transformation the equation of the surface has been reduced to a second order determinant equated to zero, as was done by the author for the generic case in a paper appearing in the Rendiconti. The surface has *27* double-lines of which 15 are denoted as regular double-lines, 5 in each of the six flats of the hexad, and 12 accessory double-lines, two in each flat which intersect the five regular double-lines belonging to the flat. These accessory double-lines are tac-loci on the $V_4^{(4)}$. The necessary and sufficient conditions that the surface shall be self-dual are also found. This surface has six nodal quadrics, one in each flat. (Received March 4, 1931.)

151. Professor L. M. Graves: *On an existence theorem of the calculus of variation.*

Hahn has given a rather general existence theorem for the minimum of a semi-definite integral in the plane. (See Wiener Berichte, vol. 134 (1925) pp. 437–447.) In making the proof he has recourse to a geometric form of argument which is not extensible to integrals in space of three or more dimensions. To fill this gap, the following lemma, which holds for any number of dimensions, is proved. Let the sequence of rectifiable curves C_n converge to a single point X_0, and suppose that the Weierstrass function $E(x_0, x', X')\geqq 0$ for all directions x' and X', while $E(x, x', -x')>0$ for all directions x'. Then there exists a positive number m such that $\limsup (JC_n)\geqq m \limsup L(C_n)$, $\liminf J(C_n)\geqq m \liminf L(C_n)$. Here $J(C)$ denotes the value of the integral $\int F(x, x')dt$ along the curve C, while $L(C)$ denotes the length of C. For the case of two dimensions, a much simpler proof than Hahn's is indicated. (Received March 4, 1931.)

152. Mr. J. M. Feld: *Analytic curve for which the chord equals the arc.*

A proof is given that curves in three-space, such that the chord OP equals the arc OP for every point P, lie in minimal planes. This result was announced by E. Kasner at the thirty-first annual meeting of the Society. An investigation of curves having the same property in four-space is also made. (Received March 4, 1931).

153. Mr. J. M. Feld: *The generalized pedal transformation.*

If on the tangents t to a plane curve K points P are found such that the cross-ratio of PA, PB, PC and t be constant, A, B, and C being three fixed non-collinear points, the locus of P is defined as the generalized pedal of K with respect to A, B, C. If B, and C are the circle points at infinity then AP will make a constant angle with t, the magnitude of which is determined by the fixed cross-ratio. The generalized pedal transformation is proved to be a contact transformation. It is shown that W-curves are their own generalized pedals and a number of properties of the transformation are determined. (Received March 4, 1931.)

154. Mr. J. M. Feld: *Birational contact transformations.*

A proof is given that a contact transformation in the plane is birational if and only if its directrix equation has the following property: that if either set of its variables be regarded as parameters of the directrix equation represent a homoloidal family of curves. Birational contact transformations are shown to be equivalent to a sequence of transformations, the first a Cremona transformation, then a polarity and finally, another Cremona transformation. Homogeneous coordinates are used throughout. (Received March 4, 1931.)

155. Professor B. A. Bernstein: *On unit-zero Boolean representations of operations and relations.*

In a previous paper the author (in cooperation with Mr. N. Debely) made basic notion of *unit-zero functions* in a method of obtaining arithmetic representations of arbitrary operations and relations in a finite class of elements. The present paper determines to what extent this method can be used analogously to obtain *Boolean* representations of arbitrary operations and relations. (Received February 11, 1931.)

156. Professor B. A. Bernstein: *Note on the condition that a Boolean equation have a unique solution.*

Whitehead has obtained certain results concerning the condition that a Boolean equation have a unique solution. Professor Bernstein obtains these results much more simply than does Professor Whitehead, and he also points out the geometry underlying them. (Received February 11, 1931.)

157. Dr. Leonard Carlitz: *A problem in additive arithmetic.*

An asymptotic expression is found for $\sum\alpha(n_1)\alpha(n_2)\cdots\alpha(n_\nu)$, the summation extending over all the partitions of a large integer n into ν parts ($\nu\geqq 3$), $\alpha(n)$ satisfying certain conditions. The method is an application of the Hardy-Littlewood analysis, particularly as applied by them to the Goldbach Problem. The paper will appear in the Quarterly Journal of Mathematics. (Received March 2, 1931.)

158. Dr. Leonard Carlitz: *A class of algebraic fields of characteristic p.*

The binomial equation $x^m = M$ over the field of rational functions with coefficients in a Galois field of order p^n, where m is a divisor of p^n-1, defines

a field resembling in many particulars the ordinary circular fields. Its properties are here developed in a very simple way; this is especially true of the theory of units. (Received March 2, 1931.)

159. Dr. Leonard Carlitz: *Quadratic fields over the field of rational functions, modulo* 2.

Following Artin (Mathematische Zeitschrift, vol. 19 (1924), pp. 154–246), who limited himself to the case of an odd prime modulus, a treatment of the elementary and analytic ideal-theory of the fields described is given. An interesting feature is that the number of fields having the same discriminant is infinite. (Received March 2, 1931.)

160. Dr. Leonard Carlitz: *New Diophantine automorphisms.*

Applying a simple principle, it is indicated how an unlimited number of Diophantine automorphisms may be derived from certain invariants of binary forms of odd order. Eisenstein's quaternary quartic appears as a special case. (Received March 2, 1931.)

161. Professor Morgan Ward: *Systems of Appell polynomials.*

The Appell polynomials are generalized by considering sets of M polynomials $P_r^{(N)}(x)$, $(r=1, \cdots, N)$ of degrees $N=0, 1, 2, \cdots$, satisfying the relations

$$\frac{dP_r^{(N)}(x)}{dx} = \sum_{j=1}^{M} d_{rj} P_j^{(N-1)}(x), \qquad (N = 1, 2, \cdots),$$

where the d_{rj} are constants independent of N and x. The formal algebraic theory of the Appell polynomials is shown to carry over to these sets of polynomials with only slight modifications. (Received March 6, 1931.)

162. Professor Morgan Ward: *Some arithmetical properties of sequences satisfying a linear recursion relation.*

The author gives a number of congruences to a prime modulus satisfied by solutions of a linear difference equation with constant coefficients when its characteristic equation is irreducible with respect to the modulus. (Received March 6, 1931.)

163. Professor Morgan Ward: *Conditions for the solubility of the diophantine equation.*

In this paper the author obtains conditions that the diophantine equation $x^2 - N^2Dy^2 = -1$ be soluble not depending on continued fraction theory. It is sufficient to consider the equation when N is an odd prime P and D is square-free and prime to P. If D is a non-residue of P, a necessary and sufficient condition that $x^2 - P^2Dy^2 = -1$ be soluble is that $x^2 - Dy^2 = -1$ be soluble. If D is a residue of P, this condition though necessary is not sufficient for solubility. A second necessary condition is stated in terms of the quadratic character of the number $u+iv$ with respect to the number $a+ib$ in the field $F(i=\sqrt{-1})$, where (u, v) is the least solution of $x^2 - Dy^2 = -1$ and the norm of $a+bi$ is P.

In case P is of the form $8n+5$, these two conditions are sufficient for solubility. Application is made to the equation $x^2-5P^2y^2=-1$. (Received March 6, 1931.)

164. Professor Morgan Ward: *Orthogonal and periodic systems of Appell Polynomials.*

The work of the previous paper on Systems of Appell polynomials is extended by subjecting the polynomials to additional restrictions which generalize the orthogonal and periodic properties of the ordinary Appell polynomials. In particular, the periodic properties of Appell polynomials recently developed by the author (Annals, vol. 31 (1930) pp. 43–51) are extensively generalized. (Received March 6, 1931.)

165. Professor Morgan Ward: *Orthogonal Appell polynomials.*

It is shown that under certain restrictions as to convergence, the only Appell polynomials which are orthogonal for some range are substantially the Hermite polynomials. (Received March 6, 1931.)

166. Professor Morgan Ward: *The linear form of numbers represented by a homogeneous polynomial in any number of variables.*

Necessary conditions are obtained that all of the numbers properly represented by a homogeneous polynomial in any number of variables may be of the linear forms nz, $nz+a_1$, $\cdots$ or $nz+a_r$ where n is any integer of $a_1, \cdots, a_r$ are r distinct integers less than n and prime to it. Application is made to the problem of obtaining binary forms such that the prime factors of all numbers properly represented by the forms are either of the type $nz\pm1$ or divisors of n. (Received March 6, 1931.)

167. Professor W. M. Whyburn: *On the Lebesgue integral.*

It is shown that a necessary and sufficient condition for a bounded function $f(x)$ on $X: a\leqq x\leqq b$ to be measurable on X is that there exists a sequence of *simple functions* (*horizontal* or *step functions*) which approaches $f(x)$ almost everywhere on X. It follows from this that the treatment given by Friedrich Riesz in Acta Mathematica, Volume 42, pp. 191–205, yields an integral which is identical with the Lebesgue integral. (Received March 3, 1931.)

168. Dr. Leo Zippin: *Generalization of a theorem due to C. M. Cleveland.*

At the suggestion of Professor R. L. Moore a plane theorem recently announced by Cleveland (Abstract 36-9-331) is generalized to euclidean spaces of dimension n greater than 2. It is proven that if C is a continuous curve nondense in E_n, $n>2$, and B is a closed and totally disconnected subset of E_n such that $B''=B\cdot C$ does not separate (locally separate) C, then there exists a ray L in E_n such that 1) $L>B$, 2) $L\cdot C=B^0$ is totally disconnected, 3) B^0 does not separate (locally separate) C, 4) $B^0-B<C^{n-1}$, where C^{n-1} denotes the points of C which are of dimension at least $n-1$ in C, 5) the endpoint of L is an arbi-

trary point of B, 6) if B is bounded, L is an arc with arbitrary endpoints in B. It is proven also that on the hypothesis above, there exists an acyclic continuous curve T of E_n with properties analogous to those of L and such that B is the set of endpoints of T. Certain results of Moore and Zarankiewicz relating to paths that do not separate a given continuous curve, and to accessibility of points of one dimensional continua in 3-space, respectively, are given considerable generalization. (Received February 12, 1931.)

169. Professor J. H. Roberts: *A property related to completeness.*

In December, 1926, R. L. Moore presented an axiom 1′ (see abstract No. 23, this Bulletin, vol. 33, p. 141) which gives a property resembling completeness. However, Moore has given an example of a space satisfying his axiom 1′ yet which is not metric and, therefore, not complete. He raised this question: *Is every metric space satisfying Axiom 1′ a complete metric space?* The present paper answers this question in the affirmative. (Received March 14, 1931.)

170. Dr. D. H. Lehmer: *Arithmetic periodicity of Bessel functions.*

The object of this paper is to study the arithmetical properties of the solutions of the difference equation $U_{n+1}=(an+b)U_n+cU_{n-1}$, where a, b, and c are integers. This equation has for special cases the equations $U_{n+1}=nU_n$ and $U_{n+1}=bU_n+cU_{n-1}$ on which the topics allied to Wilson's theorem and Lucas' functions are respectively founded. The complete solution of the given difference equation is given in terms of Bessel's functions and most simply by Lommel's polynomials. The chief aim of the investigation is to determine the period of the general solution U_n taken with respect to a modulus M, any number prime to a. The principal theorem is as follows: If ac is prime to M and if e is the exponent to w which c belongs modulo M, then the proper period of $U_n \pmod M$ is the product of the L.C.M. of e and M by some divisor of twice their G.C.D. Examples show that these limits for the period cannot be made sharper. (Received March 10, 1931.)

171. Dr. D. H. Lehmer: *On the arithmetic of double series.*

This investigation is concerned with the properties of sums of the type $h(n)=\sum f(i)g(j)$, where f and g are numerical functions and where i and j range over all positive integral solutions of $\psi(i, j)=n$. The function $\psi(i, j)$ is a single and integral valued function and determines a method of grouping the terms of a double series. The cases $\psi(i, j)=i+j-1$ and ij are well known and give rise to Cauchy and Dirichlet multiplication. The general symbolic product $h(n)$ is called ψ-multiplication. A sytem of 5 conditions on ψ is found to be necessary and sufficient to render ψ-multiplication finite, associative, commutative, and to insure the existence of a single unit of multiplication, and unique quotient when division is possible. The existence of infinite series expansions, whose connection with ψ is the same as that of power and Dirichlet series with the special cases $\psi(i, j)=i+j-1$ and ij, is shown to depend upon the solutions of certain functional equations. Several examples and special cases are considered. (Received March 10, 1931.)

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OFFICIAL COMMUNICATIONS

Meetings of the Society have been fixed at the following times and places:

NEW YORK CITY, April 3-4, 1931.

Abstracts must be in the hands of the Secretary of the Society, Professor R. G. D. Richardson, 501 West 116th St., New York City, not later than March 13. There will be a symposium on the theory of groups in which Professors H. F. Blichfeldt, L. P. Eisenhart, W. A. Manning, and others will participate. In order to be printed in the Bulletin in advance of the meeting, the abstracts must be in the hands of the Secretary not later than March 6.

CHICAGO, April 3-4, 1931.

Abstracts must be in the hands of Associate Secretary Mark H. Ingraham, University of Wisconsin, Madison, Wisconsin, not later than March 12. In order to be printed in the Bulletin in advance of the meeting, the abstracts must be in the hands of the Associate Secretary not later than March 6. By invitation of the program committee, Professor G. Y. Rainich will deliver an address entitled *Some recent applications of mathematics to physics.*

THE UNIVERSITY OF CALIFORNIA, BERKELEY, CALIFORNIA, April 11, 1931.

Abstracts must be in the hands of Associate Secretary T. M. Putnam, University of California, Berkeley, California, not later than March 21. In order to be printed in the Bulletin in advance of the meeting, the abstracts must be in the hands of the Associate Secretary not later than March 6.

R. G. D. RICHARDSON, *Secretary of the Society.*

Articles for insertion in the BULLETIN should be addressed to E. R. HEDRICK, Editor of the BULLETIN, University of California at Los Angeles. Reviews should be sent to W. R. LONGLEY, Yale University New Haven, Conn. Notes should be sent to H. W. KUHN, Ohio State University, Columbus, Ohio.

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The initiation fees and the annual dues of members of the Society (see this Bulletin, p. 322, May, 1930), are payable to the Treasurer of the Society, G. W. Mullins, 501 West 116th St., New York.

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